The Employment Cost of Sovereign Default*

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Abstract

This paper analyzes the interaction between government default decisions and labor market outcomes in an environment with persistent unemployment and financial frictions. In the model, sovereign risk worsens the conditions for firms to pre-finance production and vacancies. This generates anticipation effects of default risk and a new type of endogenous domestic default cost — the employment cost of default. The model is estimated to match the Portuguese debt crisis of 2012. The qualitative and quantitative performance of the model hinges on labor market frictions responsible for the persistence and asymmetry of default costs. Quantifying the costs associated with the Portuguese debt crisis yields a 2.1% lower bound on the quarterly consumption loss. A counterfactual debt default would have resulted in persistent welfare losses and higher spreads, even in resolution of the debt crisis. Introducing labor policies affects a government's ability to commit to debt repayment, which largely impairs cost-reducing effects.

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1 Introduction

When a country encounters problems in repaying its external debt obligations, the key question for policy makers is typically how to reduce the economic costs associated with such crises. Since most debt crises occur at the same time as massive recessions, a growing empirical and theoretical literature studying these costs focuses on output losses.[1] In order to devise policies that can address or alleviate the economic problems arising during these episodes, we need a better understanding of the underlying factors that lead to output drops. In this paper, we argue that labor is crucial for understanding the full scope of the economic costs associated with debt and default crises, and we examine public policy options that relate to the labor market, their efficacy in addressing unemployment, and how various options interact with optimal debt and default policies.

When the sovereign debt crises erupted in the Euro Area in early 2010, unemployment rates rose to unprecedented levels. One contributing factor was that concurrent with rising bond spreads, transitions from unemployment to employment slowed, as fewer people searched for work successfully. Furthermore, job losses increased, with employment-to-unemployment transitions spiking at around the same time as sovereign spreads. It also appears that job vacancies became more scarce. Notably, these developments could be observed in troubled countries both where the crisis culminated in a haircut (Greece) and where debt default was avoided (Portugal, Spain, Italy, Ireland.)

Understanding and analyzing this empirical pattern requires a framework that allows for an endogenous level of involuntary unemployment and for job mobility, which reacts to borrowing risk even if debt is repaid ex post. In other words, the empirical regularities call for the inclusion of anticipation effects in models that try to capture debt crisis episodes. But implementing such anticipation effects poses a series of theoretical challenges because it requires a dynamic and forward-looking cost structure. In fact, the majority of models in the quantitative default literature impose exogenous or endogenous costs that only materialize after an actual default. Here we propose a theory that can accommodate anticipation effects in order not only to capture the empirics of default more closely, but also to quantify the cost of default *risk* and investigate a different set of counterfactual policies that address the labor market directly. We rely on three results that make the model tractable for estimation.

This paper introduces labor market frictions à la Diamond, Mortensen and Pis-

[1] Empirical work on the output loss of default includes for example Yeyati and Panizza (2011) and Furceri and Zdzienicka (2012).

sarides[2] that interact with financial frictions in a default model in the tradition of Eaton and Gersovitz (1981). In the model, the government chooses debt but lacks commitment to repayment, allowing for the possibility of debt default. Heterogeneous firms hire workers subject to matching frictions and borrow to pre-finance an idiosyncratic share of production and vacancies. Workers are either employed or unemployed and search on and off the job for employment opportunities. Job losses are triggered by both aggregate productivity shocks and heterogeneous firm financing shocks, and labor mobility is constrained by search and matching frictions. The government understands that its tax, debt and default decisions affect labor market outcomes during and in anticipation of default due to the forward-looking optimizing behavior of the private sector. The model therefore features a dynamic strategic game between the government and private agents.

The main theoretical contribution is that the model endogenously creates an employment cost of sovereign default both in anticipation and after the realization of a debt default. When sovereign bond prices fall due to rising default risk, firms' financing conditions deteriorate. As a consequence, firms post fewer vacancies and can become debt-constrained, which forces them to let go of workers.[3] The wage setting protocol does not prevent such match dissolution, and workers face the uninsurable idiosyncratic income risk of becoming unemployed. Higher unemployment reduces production and deepens the recession. This is consistent with evidence showing that a decline in labor input significantly contributed to GDP drops during the European debt crises (Wright (2014)). The resulting rise in unemployment therefore creates an endogenous cost of default, and importantly, an actual cost of default *risk*.

The combination of endogenous default costs and anticipation effects allows us to conduct three distinct exercises. First, in order to tease out the importance of labor market frictions, we analyze those model ingredients that are qualitatively and quantitatively crucial for the performance of the model. Second, we use an estimated version of the model to quantify the cost of sovereign default risk on workers during the Portuguese debt crisis. Third, we conduct policy counterfactuals that contrast debt crises with default crises and show how different labor market policies can affect a government's ability to borrow and repay debt credibly.

Both the persistence and the asymmetry of the employment cost of default are central

[2] See e.g. Rogerson, Shimer, and Wright (2005) for a survey.

[3] Firm-level evidence documents the fact that sovereign risk depresses job vacancies and employment growth via bank lending (Acharya, Eisert, Eufinger, and Hirsch (2018)).

to the model's qualitative ability to generate debt default in equilibrium, as well as to its quantitative ability to replicate plausible debt-to-GDP ratios at realistic default probabilities. The latter is a notoriously difficult task because high default costs allow for borrowing large amounts and typically generate high levels of indebtedness, but simultaneously deter the sovereign from defaulting, leading to too low default rates. Here, the persistence in unemployment resulting from matching frictions prolongs the default cost and so increases average indebtedness. At the same time, the asymmetry of the employment cost, stemming from the fact that unemployment shoots up more when default occurs in high employment or high productivity states, addresses the problem of how simultaneously to generate plausible default frequencies (Aguiar, Chatterjee, Cole, and Stangebye (2016)). The asymmetry allows the sovereign to borrow extensively and cheaply in high productivity states but also makes default optimal after sudden productivity reversals, and so increases default frequency. Cost asymmetry hinges on the exit of firms that become debt-constrained during default, while the model cannot sustain any debt-default dynamics solely through more costly firm entry. Replacing the employment cost of default with output costs cannot address labor market dynamics and fails to replicate a positive correlation between private and public interest rates.

We estimate the model on Portuguese data using simulated method of moments, and find a good fit with tight asymptotic standard errors on parameter estimates. Subsequent model simulations can replicate the non-targeted dynamics of labor market variables around the Portuguese debt crisis. For instance, they accurately capture the joint rise in the sovereign interest rate spread and the unemployment rate, and correctly predict the change in labor mobility in and out of jobs. Next, we quantify how much households suffered from the crisis, i.e. how costly the *anticipation* of default was, irrespective of the fact that it was ultimately averted. This requires isolating the recessionary effect of default risk from other fundamentals. Our main finding indicates that default risk triggered a quarterly consumption loss for an employed worker of at least 2.1% at the peak of Portugal's debt crisis.

In a counterfactual scenario assuming that the Portuguese government had defaulted on its debt obligations in 2012, we find significant welfare losses not only on impact (about 6%) but also in the resolution of the crisis. Persistently worse labor market conditions prevent the counterfactual economy from bouncing back quickly despite large capital inflows, and are responsible for higher sovereign spreads and lower consumption levels. This cautions against political prescriptions favoring more timely defaults at a short-run cost, in the hope of achieving better economic conditions such as lower interest rates sooner.

Finally, we consider whether other labor market policies could have addressed the hardships faced by Portuguese workers more directly. For instance, we look into policies that facilitate matching, increase unemployment benefits, reduce the price of vacancies and lower job destruction rates. We find that labor friction-alleviating policies reduce the consumption cost from default risk and default probabilities only moderately when the government and private agents reoptimize policies. The reason is that the government's ability to commit to debt repayment changes. This indicates that taking into account general equilibrium effects can have important implications for the effectiveness of labor policies that try to address fiscal borrowing crises.

Many empirical studies have looked at the link between default events and the worsening of the real sector. In particular, evidence has accumulated that non-financial firms are dependent on external finance and that their spreads are highly correlated with sovereign rates.[4] Yet the interplay between public borrowing crises and the labor market has received little attention.[5] Such imbalance seems at odds with the obvious hardships caused by rising unemployment levels. Moreover, failing to study the interplay between default and the labor market may prevent identification of means to alleviate fiscal crises by targeting the labor market directly. This study shows that the slow recovery of the labor market contributes significantly to the costs endured by economies facing sovereign risk.

Part of the contribution of this paper is to incorporate a labor market, in terms of a fully specified dynamic search and matching framework with an endogenous wage setting, into the default literature. In doing so, this work relates to recent studies modeling labor frictions in an optimal default framework. While Na, Schmitt-Grohé, Uribe, and Yue (2018) and Bianchi, Ottonello, and Presno (2020) assume downward wage rigidity and Balke and Ravn (2016) assume a static matching function, we believe this paper is the first to merge the default literature with the dynamic search-theoretical approach of modeling labor markets. One advantage is the possibility to study a wider

[4]See e.g., Bai and Wei (2012); Avino and Cotter (2014); Bedendo and Colla (2015); Kaas, Mellert, and Scholl (2020).

[5]One exception is Acharya, Eisert, Eufinger, and Hirsch (2018), who present evidence that the loan contraction triggered by the European debt crisis depressed job creation and employment growth.

range of policy tools. To start, since inefficiencies cannot easily be undone by a policy maker, unemployment remains a concern in equilibrium, especially during default. In contrast, in a setup with downward wage rigidity and flexible exchange rates, default is associated with zero unemployment. More generally, in standard sticky wage New Keynesian models, an increase in unemployment benefits has no effects, regardless of the stance of monetary policy (Christiano, Eichenbaum, and Trabandt (2016)). Moving to a search and matching approach thus has the advantage of allowing us to study changes in unemployment benefits and policies that increase the efficiency of vacancy creation and job formation.

Since this paper proposes a channel through which default triggers a rise in unemployment, it adds a novel source of endogenous default cost to the quantitative default literature, surveyed in Aguiar, Chatterjee, Cole, and Stangebye (2016). This literature aims at explaining why sovereign governments repay debt despite their lack of commitment, and at replacing the ad-hoc exogenous punishment a government endures in case of default with an endogenous mechanism. The proposed employment cost of default complements the trade exclusion cost in Mendoza and Yue (2012), who first endogenized the default penalty by assuming that default impairs the import of foreign intermediate goods. A number of other papers model the endogenous connection between sovereign default and the private sector through financial intermediation.[6] Our research shares with these papers the link between sovereign and private financial conditions, but with two important differences. First, we highlight how this link further transmits to the labor market. This allows us to study the devastating effects of debt crises on unemployment levels and to look at policies that aim to provide relief directly to the working population. Second, in those papers, output is not reduced by an increase in the riskiness of lending to fiscal authorities, but rather by the actual realization of defaults. Thus the implications for the timing of events are very different from ours.

The distinctions between the present analysis and those other models are important for a number of reasons. To start, unlike the aforementioned papers, our work can reconcile findings by Yeyati and Panizza (2011), which indicate that, contrary to what is typically assumed, output contracts and unemployment tends to rise even in the period leading up to a default. This is in line with European experiences during the debt crises, when unemployment rates soared at the mere anticipation of default, even

[6]See e.g., Perez (2015); Engler and Steffen (2016); Niemann and Pichler (2017); Sosa-Padilla (2018).

in countries that did not default ex-post. On a theoretical note, the framework allows us to disentangle the recessionary effect of default risk from the adverse output effects of other factors. We can thus ask to what degree consumption drops in the European debt crises were due to sovereign risk versus other adverse shocks. Two things are key in analyzing this question: First, default risk endogenously transmits to the productive sector. Second, the private sector must be forward-looking, because otherwise, firms would not adjust production in anticipation of default.

Since the model economy is prone to expectation-driven crises, the paper also contributes to studies on equilibrium multiplicity in default models, most closely Detragiache (1996), Balke (2018) and Galli (2021). Intuitively, if lenders hold pessimistic beliefs about the likelihood of a future default, they will charge a high risk premium on debt that in turn raises unemployment levels and the incentives for a sovereign to default. This allows for pessimistic lenders' expectations to become consistent in equilibrium. While these papers share the intuition of self-fulfilling prophecies, the novel feature is that we include such a mechanism into an infinite horizon framework that can be quantitatively estimated. The key departure from the standard framework with a unique equilibrium (Auclert and Rognlie (2016)) is the inclusion of a third state variable, but the setup retains the standard timing, auction design and maturity structure of debt, in contrast to other studies on multiplicity[7].

The remainder of the paper is organized as follows. Section 2 presents key empirical facts about labor market variables during debt crises. Section 3 introduces the model and characterizes the equilibrium. Section 4 outlines our empirical strategy. Section 5 discusses the main results relating to the employment cost of default and the quantification of default risk. Section 6 considers policy counterfactuals. Section 7 concludes.

2 Labor markets during debt crises

This section motivates our study of the crucial role that labor plays in accounting for the full scope of the economic costs associated with debt and default crises, by establishing key facts about the labor market during the European debt crisis of the early 2010s. A particularly concerning feature of this episode was soaring unemployment, not only due to the often persistent negative effects of unemployment spells on the working age

[7]See Cole and Kehoe (2000); Aguiar, Chatterjee, Cole, and Stangebye (2019); Conesa and Kehoe (2017); Calvo (1988); Ayres, Navarro, Nicolini, and Teles (2018); Lorenzoni and Werning (2019); Aguiar and Amador (2020); Stangebye (2020).

Table 1: Growth decomposition 2008-2013

	Portugal	Spain	Italy	Ireland	Greece
GDP	−7.7	−7.9	−8.6	−5.7	−30.1
Contribution					
1 Total labor	−10.4	−12.5	−6.6	−14.7	−9.2
2 Total capital	4.4	5.0	1.6	8.9	1.9
2.1 ICT capital	1.5	1.7	0.9	1.9	1.6
2.2 Non-ICT capital	2.9	3.4	0.6	7.0	0.3
3 TFP growth	−1.8	−0.5	−3.8	3.1	−23.7

Notes: GDP growth in constant prices; all other variables measure the percentage point contribution to annual GDP growth. Data: OECD.

population, but also because of their unequal impact across the income distribution. Yet despite such distributional welfare effects, a large body of empirical literature on the cost of default has focused on output losses.[8]

While GDP growth slowed during the European debt crises, labor services were indeed an important, if not the most important, source for observed output declines. This is somewhat puzzling, as standard theory predicts that households may react to crises with an increase in labor supply. Besides, one may suspect capital, and not labor, to be the driver of slowing production, for example through cut-backs in investment. The growth decomposition of output in Table 1 reveals that on average, total labor services contributed most strongly to the GDP losses experienced by affected countries. In comparison, the contribution of total factor productivity was in most countries an order of magnitude smaller than that of labor, and capital did not account for any of the output losses. In fact capital contributed positively to output.[9]

The question is then what aspects of labor services are responsible for their strong adverse effects on output during debt crises. To start, Figure 1 plots the sovereign bond spreads of Portugal, Spain, Greece, Italy and Ireland over German 10-year bonds together with quarterly, seasonally adjusted unemployment rates. These countries were confronted with serious fiscal problems that led to sharp increases in bond spreads, which culminated in the Greek haircut on government bonds in 2012. At the same time, these nations faced immense increases in their unemployment rates. The time lines thus

[8] See e.g. Yeyati and Panizza (2011) and Furceri and Zdzienicka (2012).

[9] The growth decomposition uses annual data provided by the OECD. A similar argument based on data from the Conference Board Total Economy Database has been made by Wright (2014).

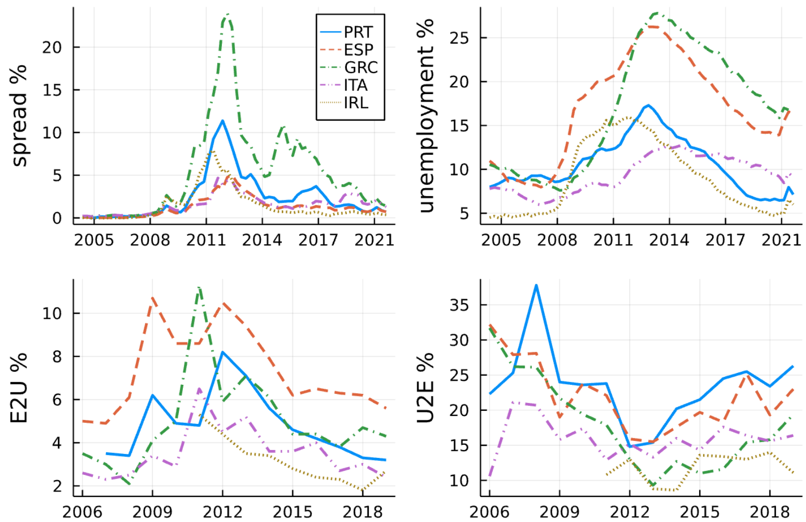

Figure 1: Sovereign spreads and unemployment, E2U and U2E rate.

visualize the fact that sovereign bond spreads are highly correlated with unemployment rates, as documented in Table 2.

A natural starting point to explain these massive increases in unemployment during the European debt crises is to analyze key drivers of the labor market. To this end, Figure 1 shows rising borrowing costs alongside average job loss probability as well as the job-finding rate of unemployed workers for the same set of troubled countries. The graphs highlight that employment-to-unemployment (E2U) transitions spike at around the same time as sovereign spreads, while unemployment-to-employment (U2E) transitions simultaneously slow down. Both effects add to rising unemployment rates. Furthermore, it appears that job vacancies become more scarce. The corresponding correlations in Table 2 indicate that elevated default risk is associated with higher E2U rates, worse U2E probabilities and fewer vacancies. Hence, changes in labor market mobility as well as the vacancy-posting behavior of firms seem crucial in accounting for the persistent unemployment dynamics of sovereign debt crises and their costs. We believe this is the first paper that attempts to model these features of the labor market in the context of a sovereign default model.

Finally, it is noteworthy that the documented empirical regularities hold up when countries face rising borrowing costs in anticipation of a future default, even if the debt default can be avoided ex-post. That means that there is ample evidence that in times of higher sovereign default risk, unemployment tends to be higher, independent of the

Table 2: Sovereign spreads: Correlation with labor market variables.

Country	cor(spr,u)	cor(spr,E2U)	cor(spr,U2E)	cor(spr,v)
Portugal	0.704	0.644	−0.505	−0.385
Spain	0.823	0.745	−0.670	−0.355
Greece	0.671	0.664	−0.621	−0.616
Italy	0.562	0.739	−0.337	−0.348
Ireland	0.822	0.946	−0.221	−0.415

Notes: Data: OECD, Eurostat, ECB, ISTAT; see Section 4 for details.

realization of an actual default. This distinction is important because it may lead to very different policy implications than those that a theory without such anticipation effects would imply. In fact, most sovereign default models impose exogenous or endogenous welfare costs only once a default actually materializes. In contrast, this work proposes a theory that can accommodate such anticipation effects and allow a different set of counterfactual policies to be investigated.

3 Model

3.1 Preferences and technology

We consider a small, open economy in discrete time, with a benevolent government, domestic workers and firms, and foreign lenders. The government lacks commitment to repay the debt it sells to lenders, allowing for the possibility of debt default. Workers are either employed or unemployed and search for jobs in a frictional labor market. Firms face idiosyncratic and stochastic financing needs and post job vacancies to attract workers.

Workers. There is a unit measure of infinitely-lived and risk-averse workers indexed by i. Workers derive utility from consumption according to their preferences $\mathbf{u}(c_{it})$ and discount future utility with β. In each period t, a share $(1-u_t)$ of workers is employed and consumes after-tax wage, $c_{it} = w_{it} - \tau_{it}$. The remaining share u_t of workers is unemployed and consumes unemployment benefits, $c_{it} = b_t$. We follow the standard assumption in the sovereign default literature that a government borrows and saves on behalf of workers who cannot save themselves.

Firms. Firms produce output with constant-returns-to-scale technology when they are matched with workers. The output produced in each firm-worker match is equal to aggregate productivity z_t, which is subject to exogenous shocks ϵ_t and follows a stochastic AR(1) process in logs:

$$\log(z_{t+1}) = \rho \log(z_t) + \sigma_\epsilon \epsilon_{t+1}. \tag{1}$$

In each period t, firm j has to pre-finance the stochastic share k_{jt} of output before production. k_{jt} is i.i.d. and drawn from a continuous uniform distribution $U[0,1]$. These financing needs introduce heterogeneity among firms and capture the dependence of firms on external finance (similar to the working capital requirements in Christiano and Eichenbaum (1992) and Neumeyer and Perri (2005)). As is standard in that literature, we assume that each firm borrows at a risk-free corporate interest rate R_t within each period, but that a firm's debt burden cannot exceed a certain fraction ϕ of its output. Too high debt then forces the firm to exit the market and workers to lose their jobs. Otherwise, firms produce and compensate their employees. Finally, firms are free to enter the labor market by creating vacancies v_t at a unit cost $(1+R_t)a$. If vacancies are matched with an unemployed worker, production starts in the same period.

Labor market. Both employed and unemployed workers search for job opportunities in a frictional labor market. On the one hand, we assume that an employed worker has a high search efficiency that allows her to meet a vacancy with probability one in each period. On the other hand, unemployed workers face a matching technology $M(1-N_t, v_t)$ in the tradition of Diamond (1982), Mortensen (1982) and Pissarides (1985) that depends on beginning-of-period unemployment $(1-N_t)$ and vacancies v_t. The matching technology is assumed to be Cobb-Douglas,

$$M(1-N_t, v_t) = \min\left\{\mu e^{(1-N_t)-1}(1-N_t)^\psi v_t^{1-\psi}, v_t, 1-N_t\right\}, \tag{2}$$

where $\mu e^{(1-N_t)-1}$ is the match efficiency and ψ governs the match elasticity to unemployment and vacancies. The first term captures the fact that a high number of employed workers may congest the labor market and make it difficult for unemployed individuals to match with given vacancies. Let $\lambda_t^f \equiv M(1-N_t, v_t)/v_t$ be the firm's probability of filling a vacancy with an unemployed worker, and let $\lambda_t^w \equiv M(1-N_t, v_t)/(1-N_t)$ be the worker's probability of finding a job out of unemployment.

Matches are separated either exogenously at destruction rate $\xi(z_t)$ or endogenously because over-indebted firms are forced to exit at rate $s_t = \mathbb{P}[k_{jt}R_t z_t > \phi z_t]$. The implied law of motion for beginning-of-period employment N_t is thus given by:

$$N_{t+1} = (1-\xi(z_t))(1-s_t)[N_t + M(1-N_t, v_t)] = 1 - u_t. \tag{3}$$

Firms make take-it-or-leave-it offers to unemployed workers at the time of hiring. However, incumbent and poaching firms compete with each other in Bertrand competition over employed workers who have outside offers. If a worker is indifferent between offers, we assume she stays in her previous job.

Government. The government is utilitarian and maximizes social welfare

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t [(1-u_t)\mathbf{u}(c_t^e) + u_t \mathbf{u}(b_t)], \tag{4}$$

by choosing taxes τ_{it}, unemployment benefits b_t, non-contingent one-period debt B_{t+1} and whether or not to default $d_t \in \{0,1\}$. To rule out Ponzi schemes, we impose the condition that debt is bounded by an arbitrarily large finite level $\overline{B} \geq B_{t+1}$. In the presence of risk aversion, the government would ideally equalize the consumption level within and across employment status. This possibility is limited here, i.e., while full redistribution is possible between employed workers such that each of them consumes $c_t^e = w_{it} - \tau_{it}$, unemployed workers receive only a fraction ζ of this consumption level, $b_t = \zeta c_t^e$. This constraint stands in for unmodeled incentive issues typically assumed in relation to disutility of working, costly job search processes and unemployment benefits. Furthermore, aggregate productivity evolves according to (1), and the following fiscal budget constraint needs to be satisfied:

$$(1 - u_t)c_t^e + u_t b_t = d_t[q_t B_{t+1} - B_t] + w_t, \tag{5}$$

where $w_t = \int w_{it} di$ is aggregate taxable labor income, and q_t is the price of government bonds. As detailed in Section 3.2, the strategic government faces implementability constraints because it understands how its policy affects workers', firms' and lenders' choices of u_t, w_t and q_t. Finally, the sovereign lacks commitment power to any future default, debt or tax policies.

Lenders. Lenders are risk-neutral and buy sovereign bonds to break even in expectation. Under optimality, the government bond price q_t thus equals the expected discounted marginal return

$$q_t = \frac{\mathbb{P}(d_{t+1}=1)}{1+r}, \tag{6}$$

where $\mathbb{P}(d_{t+1}=1)$ is the probability that the government will repay its debt in the following period, and r is the lenders' discount rate.

Lenders also satisfy firms' intratemporal financing needs. The lending rate to the corporate sector is given by

$$R_t = \mathcal{R}(z_t, N_t, q_t, B_{t+1}), \tag{7}$$

where the function $\mathcal{R}(\cdot)$ captures the reduced-form relation between the borrowing cost for firms and economic indicators, including sovereign default risk. A growing empirical literature supports such a relationship. For example, when sovereign spreads rise, private borrowing becomes more expensive (Adelino and Ferreira (2016); Bofondi, Carpinelli, and Sette (2018); De Marco (2019)), and numerous papers document the comovement of sovereign and corporate spreads (Bai and Wei (2012); Avino and Cotter (2014); Bedendo and Colla (2015)). While a microfoundation of the function may be of interest, we omit an explicit derivation of a link between sovereign and corporate borrowing rates in the main part of this paper but present a potential channel in Appendix A. Our formulation is based on the detrimental effects that a decline in the price of government securities has on the balance sheet of financial intermediaries, and through impairment of intermediaries' lending, on the borrowing costs of non-financial firms.[10] However, the function $\mathcal{R}(\cdot)$ also nests alternative theories of why banks hold government debt, and why sovereign default may impair their lending activity to firms. For instance, Perez (2015) models liquidity reasons as an explanation for why banks hold government debt. In Sosa-Padilla (2018), bankers trade returns to storage off against lending to the government. Chari, Dovis, and Kehoe (2020) motivate why governments may force banks to hold debt. All of these theories share an effective link between sovereign default and lending conditions in the private sector. This connection is captured by equation (7).

Timing. The timing of the model, illustrated in Figure 2, is as follows. In each period t, aggregate productivity z_t realizes first. Next, the government chooses its default d_t and debt B_{t+1} policies as well as sets taxes to determine consumption c_t^e and benefits b_t. Subsequently, lenders price sovereign and private debt, q_t and R_t. In the following stage, firms post vacancies v_t and meet workers, before observing their idiosyncratic financing needs k_{jt}. Then, firms either separate or continue to produce output, and workers consume c_{it}.

[10] This assumption is consistent with the findings in Acharya, Eisert, Eufinger, and Hirsch (2018); Bottero, Lenzu, and Mezzanotti (2020); Popov and Van Horen (2015), who show empirically that sovereign default risk negatively affects the conditions of domestic banks.

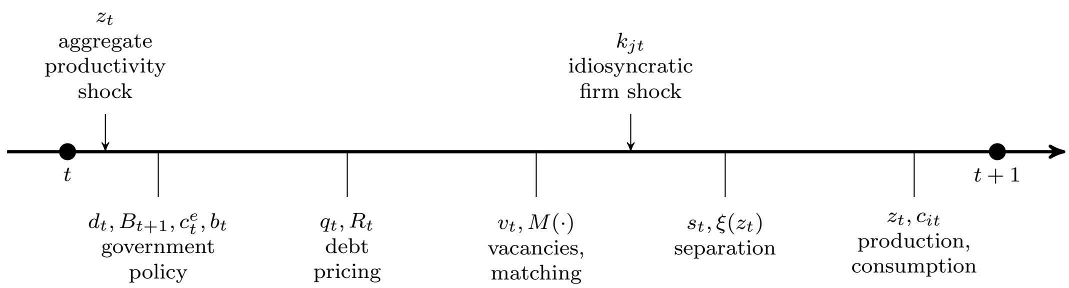

Figure 2: Timing of the model.

3.2 Recursive formulation and equilibrium

The model poses multiple challenges. To start, the government may optimally use history-dependent policies whose dimensionality grows to infinity. To make this manageable, we concentrate on Markov-perfect equilibria that can be characterized as follows. When the government chooses policies for the current period, $\delta_t = (d_t, B_{t+1}, c_t^e, b_t)$, its choices are constrained to depend only on aggregate productivity, outstanding debt and initial employment summarized by the aggregate state $\Omega_t = (z_t, B_t, N_t)$. We can thus define Markov government policy as mapping $\mathcal{D} : \Omega_t \to (d_t, B_{t+1}, c_t^e, b_t)$ and rewrite the problem recursively. Another theoretical challenge in the dynamic game is that it involves strategic and non-strategic agents: while workers, firms and lenders are non-strategic, taking δ_t, $\mathcal{D}$ and aggregate laws of motion as given, the government acts strategically, understanding how its current policies affect today's outcomes and tomorrow's state Ω_{t+1}. This means that the government is a sophisticated, large player in the dynamic game with the private sector. Finally, we have to account for the sovereign's lack of commitment. Since the incumbent government cannot dictate future public policy, we enhance the state space of welfare function $\mathcal{W}(\Omega; \mathcal{D})$ with future sovereign policy $\mathcal{D}$. This allows us to rely on a one-time deviation to find the time-consistent equilibrium. The following paragraphs explain in more detail how these considerations shape our approach for the recursive problem and equilibrium definition.

Markov-perfect equilibria allow a recursive representation of the optimization problems of workers, firms and government. Henceforth, time subscripts are omitted to denote the current period, and primes indicate the subsequent period's states and choices.

We can then write the recursive values of an employed and an unemployed worker as:

$$\mathcal{E}(\Omega;\delta,\mathcal{D}) =(1-\xi(z))\,(1-s)\,\Big[\mathbf{u}(c^e)+\beta\mathbb{E}\mathcal{E}(\Omega';\mathcal{D}(\Omega'),\mathcal{D})\Big] + (1-(1-\xi(z))(1-s))\,\Big[\mathbf{u}(b)+\beta\mathbb{E}\mathcal{U}(\Omega';\mathcal{D}(\Omega'),\mathcal{D})\Big] \tag{8}$$

$$\mathcal{U}(\Omega;\delta,\mathcal{D}) =\lambda^w\mathcal{E}(\Omega;\delta,\mathcal{D})+(1-\lambda^w)\Big[\mathbf{u}(b)+\beta\mathbb{E}\mathcal{U}(\Omega';\mathcal{D}(\Omega'),\mathcal{D})\Big]. \tag{9}$$

Similarly, a firm's expected profit and vacancy value are given by:

$$\mathcal{J}(\Omega;\delta,\mathcal{D}) =(1-\xi(z))\,(1-s)\,\Big(z-kRz-w+\beta\mathbb{E}\left[\mathcal{J}(\Omega';\mathcal{D}(\Omega'),\mathcal{D})\right]\Big) \tag{10}$$

$$\mathcal{V}(\Omega;\delta,\mathcal{D}) =\lambda^f\mathcal{J}(\Omega;\delta,\mathcal{D})-(1+R)a. \tag{11}$$

Value functions (8)-(11) are subject to constraints imposed by financing requirements (given by (7) and pinning down $k{=}\mathcal{K}(\Omega;\delta,\mathcal{D})$), the matching and separation technology (giving $s{=}\mathcal{S}(\Omega;\delta,\mathcal{D})$, $N'{=}1{-}u{=}\mathcal{N}(\Omega;\delta,\mathcal{D})$, $\lambda^w{=}\Lambda^w(\Omega;\delta,\mathcal{D})$ and $\lambda^f{=}\Lambda^f(\Omega;\delta,\mathcal{D})$), the wage setting (yielding $w{=}\mathcal{W}(\Omega;\delta,\mathcal{D})$), firms' free entry (rendering $\mathcal{V}(\Omega;\delta,\mathcal{D}){=}0$), and the evolution of productivity (1). They are indexed by current public policy δ, because private agents observe δ before they act in each period. Furthermore, private agents take as given Markov policy $\mathcal{D}$, which will be in place from the subsequent period onward.[11] Note that the probability of meeting an employed worker does not enter equation (11), because at this stage firms make identical offers in Bertrand competition, and our tie-breaking rule specifies that an incumbent worker will stay at her existing employer.

The government's problem in recursive form can be written as:

$$\mathcal{W}(\Omega;\mathcal{D}) = \max_{\delta} N\mathcal{E}(\Omega;\delta,\mathcal{D})+(1-N)\mathcal{U}(\Omega;\delta,\mathcal{D}), \tag{12}$$

subject to the fiscal budget constraint (5), redistribution constraints and the implementability constraints imposed by the optimizing behavior of the private sector, notably $N'{=}1{-}u{=}\mathcal{N}(\Omega;\delta,\mathcal{D})$, $w{=}\omega(\Omega;\delta,\mathcal{D})$, and $q{=}\mathcal{Q}(\Omega;\delta,\mathcal{D})$. The commitment constraint shows up in the fact that welfare function $\mathcal{W}(\Omega;\mathcal{D})$ is indexed by $\mathcal{D}$. The sovereign understands, however, that a subsequent period's state Ω' is affected by her choice of B' and by how δ affects N'. This sophistication allows the government to act strategically.

To define the equilibrium under lack of commitment, we proceed in two steps. First, given the aggregate state Ω and current and future government policies δ and $\mathcal{D}$, we define a private sector equilibrium. Second, we characterize the best one-period deviation on the part of the government to construct the time-consistent equilibrium.

[11] In general, the private sector can assume any sequence of future policies $(d_t^\infty, B_{t+1}^\infty, c_t^{e\infty}, b_t^\infty)$. However, since we consider Markov-perfect equilibria, it is w.l.o.g. to restrict attention to future government policies that are given by a constant function of the state $\mathcal{D}:\Omega_t\rightarrow(d_t,B_{t+1},c_t^e,b_t)$.

Definition 1. *For each state* Ω*, current government policy* δ*, and future policy function* $\mathcal{D}$*, a* ***private sector equilibrium*** *is defined as value functions* $\mathcal{E}$*,* $\mathcal{U}$*,* $\mathcal{J}$ *and* $\mathcal{V}$ *with associated policies; prices* w*,* q *and* R*; and laws of motion for* z *and* N*, such that:*

1. *the value functions solve Bellman equations (8)–(11), and the associated policies are optimal and consistent with financial constraints, the matching and separation technology and free entry;*
2. *prices* w*,* q *and* R *are the outcome of the wage setting protocol, break-even lending (6) and lenders' loan pricing (7);*
3. *employment follows (3), and productivity evolves according to (1).*

In a private sector equilibrium, the government is assumed to revert to constant policy mapping $\mathcal{D}$ after making initial choices $\delta = (B', \tau, d)$. However, due to its inability to commit, the government can neither promise constant policy mapping nor abstain from making the best decision for the current period. We therefore consider the best possible decisions of a government in a given period. Let $\delta^*(\Omega; \mathcal{D})$ be the solution to problem (12), subject to the government constraints. That means $\delta^*(\Omega; \mathcal{D})$ is the preferred policy of the incumbent government, given that its successors revert to policy $\mathcal{D}$ forever and the private sector is in equilibrium. Then a time-consistent equilibrium is characterized by the fact that this best one-time deviation coincides with continuation policy $\delta^*(\Omega; \mathcal{D}) = \mathcal{D}(\Omega)$.

Definition 2. *For each state* Ω*, a* ***time-consistent equilibrium*** *is a constant government Markov policy* $\mathcal{D}$ *and a one-time deviation policy* δ^* *such that:*

1. *given* Ω *and* $\mathcal{D}$*,* $\delta^*(\Omega; \mathcal{D})$ *maximizes welfare (12) subject to the government's budget and redistribution constraints;*
2. *given* Ω*,* δ^* *and* $\mathcal{D}$*, the private sector is in equilibrium;*
3. $\mathcal{D}$ *coincides with the preferred one-time deviation,* $\delta^*(\Omega; \mathcal{D}) = \mathcal{D}(\Omega)$*.*

This definition describes the fixed point of a dynamic game between a large, strategic government and small, non-strategic private agents. By allowing for the current policy to differ from future policies, the definition amounts to subgame perfection or time consistency of equilibrium. Therefore, the arrangement is fundamentally different from equilibria under commitment.

3.3 Characterization of equilibrium

This section discusses three key aspects of the model in equilibrium. First, we explore how default risk transmits to the labor market and how this transmission shapes the main trade-offs of the government. Second, we intuitively and formally show why multiple equilibria can arise and present our equilibrium selection mechanism. Third, we establish key results that help us achieve tractability of the model.

Policy trade-off. To understand the transmission of default risk to the labor market, it is helpful to recall that if default becomes more likely or the repayment probability decreases, then debt price q decreases due to break-even condition (6), which in turn affects loan rate R through equation (7). In addition, R also directly depends on the indebtedness of the sovereign. If public policy thus induces the loan rate to rise, more firms become debt-constrained and need to exit. This means more pre-existing jobs are destroyed and more E2U transitions take place. This mechanism directly increases unemployment. At the same time, higher borrowing rates mean higher operating costs for a firm and lower incentives to post new vacancies. Hence, if sovereign debt policy depresses job vacancies, fewer U2E transitions take place, which adds to the increase in unemployment. Through a separate fiscal channel, public policy affects wages and consumption independently of the loan rate. This follows immediately from the government budget constraint. Higher taxes directly lower the net wages of employed workers, and benefit payments to unemployed workers fall proportionally, lowering the consumption levels of both types of workers.

Since the government is strategic, it understands these connections. For instance, when the government is confronted with a low productivity shock, it trades the cost of increasing taxes to raise enough revenue for debt repayment off against the cost of increasing public debt. In the former case, higher taxes shrink net wages, and so the government sacrifices utility of workers due to lower consumption. In the latter case, a higher debt burden compromises the pre-financing abilities of firms. Hence, the sovereign sacrifices some employed workers' jobs through higher separation risk and reduces unemployed workers' probability of finding jobs. In equilibrium, defaulting becomes optimal when the utility costs from wage reductions on the employed outweigh the utility costs from higher unemployment.

This trade-off exists even absent an outright default, when the borrowing costs of the sovereign rise. As a result, the government is concerned about the fact that the labor market can be disrupted in a debt crisis, triggering the described employment costs, independently of whether default takes place or not ex-post. This anticipation effect is important for the timing of labor market adjustments before potential default events and enables the model to capture risky borrowing crises in addition to default crises.

Multiplicity. The model features multiple equilibria because the economy is prone to expectation-driven crises when lender fears become self-fulfilling. Intuitively, if lenders hold pessimistic beliefs about the likelihood of a future default, they will charge a high risk premium on debt to satisfy their expected break-even condition. Such high risk premia can then, through the interaction between financial and labor frictions discussed above, raise the unemployment level. Once a government has to make a decision whether to repay or default in a subsequent period, it faces a worse employment state, which increases pressure to default. The government's higher likelihood of default confirms initial investor fears and allows for low expectations to become consistent in equilibrium. In reverse, when lenders believe that a government will repay, high bond prices and low unemployment prevail, and the government faces no additional incentives to default. The economy thus avoids an expectation-driven crisis.

Formally, one can show that the framework admits non-unique bond price schedules due to the presence of an additional state variable, here the privately determined employment level N. To see this, start from the basic bond pricing condition (6), where we can use the Markov property of the optimal policy to replace d' with $\mathcal{D}(z', B', N')$:

$$q = \mathbb{E}\left[\frac{\mathcal{D}(z', B', N')}{1+r}\right].$$

Employment N' is determined in the private sector, and its evolution depends crucially on private sector interest rate R. Since R is a function of z, N, B' and debt price q, we can rewrite N' using $\mathtt{N}'(z, N, B', q)$:

$$q = \mathbb{E}\left[\frac{\mathcal{D}(z', B', \mathtt{N}'(z, N, B', q))}{1+r}\right].$$

This expression reveals immediately that since debt price q enters on both sides of the equation, it generally allows for more than one solution. These multiple solutions represent different bond price schedules and can give rise to multiple equilibria.

Three things are worth pointing out. First, the conditions for this multiplicity in debt price schedules hold more broadly in the presence of any additional state variable that affects default policy and is determined by the debt price. Hence, the source of multiplicity is not restricted to employment. The mechanism thus shares the intuition of self-fulfilling prophecies in Detragiache (1996), who presents a two-period model where the optimal default decision depends on what is called "policy effort", a choice variable that depends on the conditions of the debt contract and affects the probability of default in the following period.[12] Since policy effort is chosen directly by the government, its setup requires additional assumptions in order to prevent the government from committing to a certain outcome. Such an assumption is not necessary here because employment is privately determined and the government only needs to respect budget feasibility. We also incorporate such a mechanism into an infinite-horizon framework that can be quantitatively estimated.

Second, this source of multiplicity breaks the uniqueness result in the standard framework with exogenous output (Auclert and Rognlie (2016)), but is distinct from other departures studied in the literature. Specifically, it retains the standard timing (unlike Cole and Kehoe (2000); Aguiar, Chatterjee, Cole, and Stangebye (2019); Conesa and Kehoe (2017)), auction design (in contrast to Calvo (1988); Ayres, Navarro, Nicolini, and Teles (2018); Lorenzoni and Werning (2019)) and debt maturity structure (compared to Aguiar and Amador (2020); Stangebye (2020)) of the baseline framework.

Third, the occurrence of multiple pricing schedules does not necessarily imply multiple equilibria. For instance, the government could adjust its optimal debt issuance when confronted with a possibly low debt price. Instead of selling bonds at a debt auction for relatively small revenue, a better approach might be to issue fewer bonds in order to retain a better price on those that exist. This could be optimal because, ceteris paribus, any reduction in revenue from borrowing needs to be made up for with higher taxes, thus sacrificing worker consumption. The government trades these two effects off against each other, likely lowering its debt issuance at the same time as increasing its austerity measures. In order for multiplicity to arise, the trade-off must be such that the government optimally stays in the borrowing region where multiple price schedules exist. Numerically, we find that this is indeed the case in a subpart of the state space.

Given the occurrence of more than one equilibrium, we have to impose a selection criterion for our empirical analysis. Here we use the best debt price schedule with the highest employment level to determine the equilibrium that emerges. By doing this,

[12] Building on Detragiache (1996), see also Balke (2018) and Galli (2021) for two-period default models with employment and capital, respectively. Crouzet (2017) presents a dynamic framework with equilibrium multiplicity in the context of firm investment.

we conservatively limit the model's ability to create substantial default costs and force optimism to prevail. Nevertheless, sunspot-driven debt crises are a key research topic and were a critical part of the European experience (Bocola and Dovis (2019)). We thus believe that this setup may prove useful when studying fundamental- and sunspot-driven debt crises jointly in a quantitative framework.

Tractability and wages. We rely on three results to make the problem tractable for estimation. First, aggregate taxable labor income w can be expressed in closed form, avoiding the computationally costly task of solving for and aggregating individual wages w_i at each point in time.

Lemma 1. *Under the given wage setting and job search protocol, at each state $\Omega = (z, B, N)$ and for policy B', aggregate wage bill w can be expressed as*

$$w = N(1 - \xi(z))\frac{\phi}{\mathcal{R}(z, N, q, B')}z(1 - \phi).$$

Proof. See Appendix B. □

The intuition for this result is as follows. On the one hand, consider an employed worker who enters a period in a given job. This employee is able to find an outside offer due to her high search efficiency. She thus has the option to stay with her previous employer or switch to a new employer, forcing the two into competition. Bertrand competition between firms drives up the offered wage until the employee extracts the full surplus, as in Robin (2011). This means that absent a separation shock, a previously employed worker can now extract the entire output less financing cost, $w_i{=}z(1{-}k_{j(i)}R)$, where $j(i)$ is the firm j that worker i chooses. Since the worker needs to decide where to work before current financing shock k_j is revealed, she is indifferent between the two offers, and we assume that she stays in her former job.

On the other hand, consider an unemployed worker who encounters a vacancy. She receives a take-it-or-leave-it offer from the potential employer, who optimally sets a zero wage, $w_i{=}0$. The zero wage is accepted by the worker because she understands that she will receive a negative income tax credit and consume at the same level as other employees, and more importantly, that after the first period of employment she will receive an outside offer and extract a surplus.

Second, instead of having to keep track of continuation values $\mathcal{E}$,$\mathcal{U}$ and $\mathcal{J}$, all of which are altered by any change in fiscal policy, we establish that the sovereign's problem is directly recursive in welfare.

Lemma 2. *The sovereign's problem is recursive in welfare and can be expressed as*

$$\begin{aligned}\mathcal{W}(\Omega;\mathcal{D}) = \quad & \max_{\delta} \quad \mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{W}(\Omega';\mathcal{D})] \\ & s.t. \quad \mathcal{N}(\Omega;\delta,\mathcal{D})c^e + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))b = d[\mathcal{Q}(\Omega;\delta,\mathcal{D})B' - B] + \omega(\Omega;\delta,\mathcal{D}) \\ & \qquad\;\; b = \zeta c^e.\end{aligned}$$

Proof. See Appendix B. □

Lemma 2 reveals that the welfare problem is no longer a coupled Bellman equation of $\mathcal{E}$ and $\mathcal{U}$. The dynamics of the firm problem $\mathcal{J}$ become limited so as to lie within one period, because a firm receives the full surplus in the period of matching, but nothing more. Since this profit has to cover the vacancy cost, the free entry condition becomes:

$$(1+R)a = \lambda^f \mathcal{J}(\Omega;\delta,\mathcal{D}) = \lambda^f(1-\xi(z))(1-s)z(1-kR), \tag{13}$$

where k is the expected value of heterogeneous financing shock k_j, conditional on not having to separate.

Third, under mild boundedness conditions following Aguiar and Amador (2019), we establish that the Bellman equation of welfare in Lemma 2 constitutes a contraction mapping, conditional on a given bond price schedule.

Lemma 3. *The Bellman equation of welfare is a contraction, conditional on a given bond price schedule* $\mathcal{Q}(\Omega;\delta,\mathcal{D})$ *that satisfies the equilibrium selection criterion.*

Proof. See Appendix B. □

Monotonicity as part of Blackwell's sufficient conditions is demonstrated because ω and $\mathcal{N}$ depend on the continuation value only through bond price q, see Lemma 1. Hence, given bond price schedule $\mathcal{Q}$, policy δ^* stays feasible under a different continuation value and does not violate the government's budget constraint.[13] Since the Bellman operator also satisfies discounting, it is a contraction, see Theorem 3.3 in Stokey, Lucas, and Prescott (1989). Numerical solutions can then be found with common methods allowing the bond price schedule to update slowly in an outer loop.

[13] To establish a contraction result, which does not rely on a fixed bond price schedule, a dual representation of the problem can be helpful (Aguiar and Amador (2019)).

4 Empirical analysis

The model is solved on a quarterly basis using global numerical methods. The set of structural model parameters is estimated to match key moments of the Portuguese debt crisis that peaked in 2012. This section describes data sources, parameterization and the model fit.

Data. Our measures for unemployment, GDP, productivity, country indebtedness and sovereign debt spreads are all based on OECD data. The unemployment rate is defined as the number of unemployed people as a percentage of the labor force in the OECD's Labour Market Statistics. GDP per capita is taken from Quarterly National Accounts. To be consistent with the model, we use GDP per employed person from the OECD's Main Economic Indicators to construct the productivity process. Government indebtedness is captured by public sector debt as a percentage of GDP. Interest rate spreads are the difference between a country's long-term interest rate on government debt and the German counterpart. For private sector interest rates we use cost of borrowing for new, short-term loans in the MFI Interest Rate Statistics of the ECB.

Since time series on job flows are generally unavailable before 2011 on a quarterly basis, we use annual data on labor market transitions provided by Eurostat. Here, the U2E rate is defined as the total share of unemployed persons who find full-time work. The E2U mobility rate is calculated on the basis of the total number of transitions of employed persons into unemployment.

Our vacancy data stem from several sources. We use the seasonally adjusted stock of unfilled job vacancies in Portugal from the OECD. For Spain, Greece and Ireland we take Eurostat vacancy data and remove the seasonal component with quarter dummies. These time series are expressed in percentage deviations from a linear trend. Finally, we obtain the Italian job vacancy rate from ISTAT.

Estimation. To estimate the model we assume worker preferences to exhibit constant relative risk aversion, $\mathbf{u}(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and impose the functional form $\mathcal{R}(z, N, q, B') = \kappa(zN)^{\gamma_{zN}}(1 + qB')^{-\gamma_{qB}}$ on corporate interest rates. Exogenous separation follows $\xi(z) = z^{-\xi} - 1$. We thus need to estimate the set of fourteen model parameters

$$\{\sigma, \psi, \phi, \zeta, r, \rho, \sigma_\epsilon, \xi, \mu, a, \beta, \kappa, \gamma_{zN}, \gamma_{qB}\}. \tag{14}$$

Dividing this set into partitions, we apply a three-step estimation procedure. A first set of parameters $\{\sigma, \psi, \phi, \zeta\}$ is calibrated using outside estimates. The remaining

Table 3: Moments and model fit.

	data	model
average spread	0.022 (0.0018)	0.022
unemployment rate	0.10 (0.0036)	0.09
job-finding rate	0.21 (0.0013)	0.21
vacancy cost-wage ratio	0.040 (0.0018)	0.040
private annual interest rate	0.043 (0.0011)	0.040
correlation private interest-unemployment	0.52 (0.077)	0.55
correlation private-public interest rates	0.83 (0.031)	0.81

Notes: The model is solved, estimated and simulated quarterly; model moments reflect averages over time.

parameters are estimated by indirect inference. Among those, a second subset $\{r, \rho, \sigma_\epsilon\}$ can be directly matched to empirical counterparts. Finally, to find values for the majority of structural parameters $\{\xi, \mu, a, \beta, \kappa, \gamma_{zN}, \gamma_{qB}\}$ we numerically solve the full default model in equilibrium and target a number of key empirical moments. The following paragraphs discuss the details of this approach.

Regarding the first subset, parameter σ determines the intertemporal elasticity of substitution of consumption. We choose the conventional value σ=2, which is in line with a large amount of empirical evidence using either household or aggregate data. Furthermore, we let ψ=0.7. Burda and Wyplosz (1994) estimate the elasticity ψ of matches with respect to unemployment to be around 0.7 in some European countries, and Shimer (2005) uses a similar value of 0.72 for the US. Next, we use ϕ=0.1 to match corporate interest expenses. This lies at the lower bound of the estimates for Portugal in Ferrando, Blank, Neugebauer, Siedschlag, Iudice, Altomonte, Felt, and Meinen (2015), and coincides with the median interest payment burden of around 0.1 for the Euro area between 1995 and 2012, as well as interest expenses of around 6% of revenue in the US (Eckstein, Setty, and Weiss (2019)). The value for unemployment benefits ζ=0.65 targets the Portuguese replacement rate of about 65%, which is notably higher than in other European countries and the US (Esser, Ferrarini, Nelson, Palme, and Sjöberg (2013); Murtin and Robin (2018)).

In the second step, the risk-free interest rate for lenders is specified to correspond

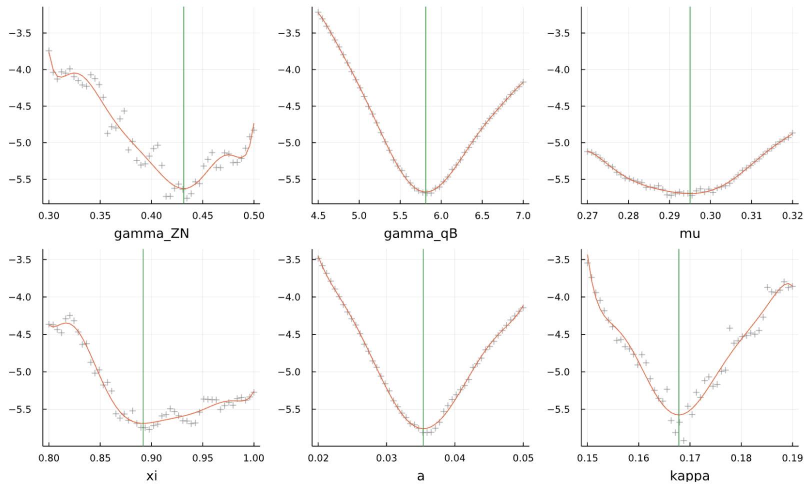

Figure 3: Parameter slices of objective function.

to the average German interest rate of 4% annually, r=0.01. To obtain estimates for productivity parameters ρ and σ_ϵ we remove a linear-quadratic time trend from our productivity series and regress the residuals on their lag in accordance with the AR(1) specification of productivity in equation (1). This results in ρ=0.71 and σ_ϵ=0.05.

Finally, we estimate the remaining parameters $\{\xi, \mu, a, \beta, \kappa, \gamma_{zN}, \gamma_{qB}\}$ by indirect inference in the third, and computationally most costly, step. We numerically solve the model at a given parameter vector, compute moments of interest from simulating the model in equilibrium and then evaluate them by means of minimum distance to their target levels. The evaluation is based on an objective function that weighs each distance by the inverse of the mean, except for the expected sovereign interest rate spread, which we match exactly using β. Figure 3 shows the slices of the objective function associated with this procedure.

The targets, summarized in Table 3, are chosen as follows. While there is no direct one-to-one mapping from parameter to moment, each parameter is discussed in relation to the target that appears particularly important in its estimation. The exogenous job destruction parameter ξ is set to match the average unemployment rate in Portugal after 1995 of about 10%. We obtain ξ=0.892. The match efficiency μ=0.295 results from targeting an average job finding rate of 21% in Portugal, taken from Murtin and Robin (2018). We follow Silva and Toledo (2009) and assume that vacancy posting costs correspond to about 4% of quarterly wages. Given this target, we derive an estimate of the vacancy posting cost of a=0.035. The main target for discount factor β is the mean

Table 4: Model parameters.

relative risk aversion	σ	2.000
match elasticity	ψ	0.700
financing parameter	ϕ	0.100
benefit replacement rate	ζ	0.650
risk-free interest rate	r	0.011
persistence of productivity	ρ	0.710
standard deviation of productivity	σ_ϵ	0.050
exogenous separation parameter	ξ	0.892 (0.04)
match efficiency	μ	0.295 (0.00)
vacancy costs	a	0.035 (0.00)
discount factor	β	0.915 (0.03)
private interest rate parameter	κ	0.167 (0.02)
interest rate elasticity	γ_{qB}	5.811 (0.93)
interest rate elasticity	γ_{zN}	0.432 (0.27)

Notes: Standard errors in brackets are computed using the asymptotic variance formula.

interest rate spread of 2.2% annually. The resulting value of 0.92 shares with most of the default literature the characteristic that the implied discount rate is counterfactually low compared to the risk-free rate. Lastly, we estimate loan rate parameters κ, γ_{zN} and γ_{qB} to target three moments related to the interest rate that corporations pay on short-term loans. First, the average short-term interest rate is 4.3% per annum. Second, interest rates and unemployment levels are highly correlated in Portugal, with a correlation coefficient of 0.52, which is much higher than in the US. Third, they are also correlated with sovereign bond spreads, showing a correlation of 0.83. Table 4 summarizes all parameter values used in the empirical section, including asymptotic standard errors for the set of estimated parameters.

Model fit. The model fit is good, see Table 3. In our estimation procedure, the model hits the target bond price spread by construction because we put a high weight on this moment. Labor market moments are also matched well: the job-finding rate as well as vacancy costs are exactly matched, and the average unemployment rate of 9% is close to its 10% target. The private sector interest rate is on average 4.0% in the model and

4.3% in the data. The interest rate positively correlates with the unemployment rate, attaining a correlation coefficient of 0.55 in the model, close to 0.52 in the data. Finally, the model matches the correlation between private loan rates and the sovereign spread within the standard error, with a value of 0.81 as compared with 0.83 in the data.

5 Results

5.1 Understanding the employment cost of default

To understand how the model endogenously generates the employment cost of default, we begin by describing equilibrium policy functions, and then turn to key model ingredients that secure success in qualitatively and quantitatively fitting the data. Figure 4 presents policy functions along debt B (x-axis) for different levels of productivity z and for employment levels about 5% below (solid) and above (dashed) mean employment. In equilibrium, welfare is decreasing in debt in the region where repayment is optimal and flat in the area of optimal default, reflecting the fact that default wipes out all inherited debt. As a consequence, other policy functions inherit abrupt kinks where optimal debt policies jump from repayment to default.[14] Evidently, default is more likely the higher the level of indebtedness, the higher the level of unemployment, and the lower the level of productivity. Close to the default threshold, spreads rise sharply and consumption is compressed below default levels. In those states it is optimal for the sovereign to sacrifice some consumption in order to avoid paying the cost of default, but if productivity or employment falls further, default becomes optimal. Firm interest rates, worker mobility and labor market tightness $\theta{=}v_t/u_t$ all move strongly with productivity and employment.

Most importantly, these policy functions give a first visualization of the employment cost of default because they show that in default, pre-financing production and vacancies becomes more expensive, workers have greater difficulty in finding jobs and labor market conditions become unfavorable. These disruptions of the real sector are the reasons why government debt becomes sustainable in the first place in this model. Moreover, pre-financing costs for firms shoot up higher when default occurs in high employment and high productivity states, showing an asymmetry in the cost structure. This rationalizes why the government is more likely to repay debt in good times.

To break down the mechanism further, we now investigate which aspects of the employment cost of default are qualitatively and quantitatively important for the model

[14] This is one of the main reasons why solving this type of model requires global solution methods.

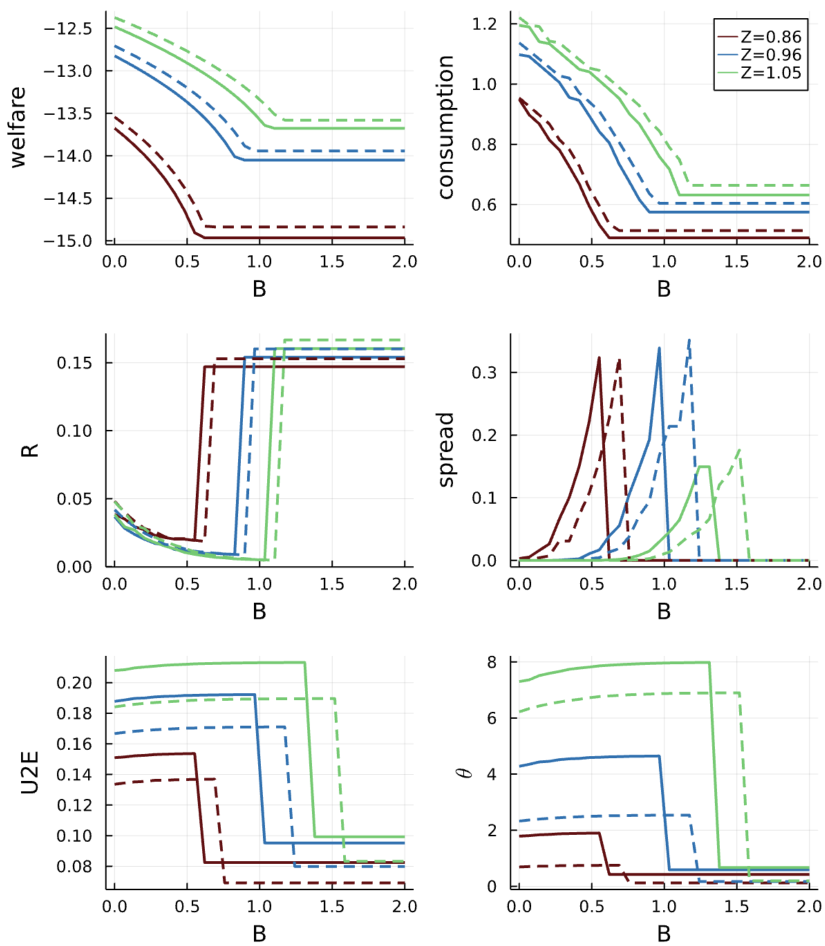

Figure 4: Policy functions for high (solid) and low (dashed) unemployment.

to perform well. In the baseline model, the sovereign's debt and default decisions affect the economy through both firm exit and entry margins, and labor market frictions prolong such impacts. As a natural starting point, we then shut down these margins one by one and compare the quantitative and qualitative performance of alternative setups with our benchmark model. Finally, we remove labor frictions altogether by replacing the employment cost of default with direct output costs, and show an intermediate case.

Table 5 summarizes the results. Next to the baseline, it reports key moments of a version of the model where default creates employment costs only on the exit margin ("E2U") and only on the entry margin ("U2E"), respectively. This exercise reveals that endogenous default costs that accelerate E2U transitions are very powerful in generating realistic moments on their own. In contrast, if default disrupted only firm entry through more expensive vacancies, it would give a very different picture: at estimated param-

Table 5: Labor frictions and type of default cost.

	baseline	E2U	U2E	output	high μ
average spread	0.022	0.022	n/a	0.0012	0.018
unemployment rate	0.09	0.09	0.09	0.00	0.04
job-finding rate	0.21	0.19	0.18	n/a	0.70
vacancy cost-wage ratio	0.040	0.040	0.040	n/a	0.038
private annual interest rate	0.040	0.040	0.81	0.026	0.043
correlation private interest-unemployment	0.55	0.55	−0.98	0.00	0.54
correlation private-public interest rates	0.81	0.81	n/a	−0.096	0.79
debt-to-GDP ratio	0.73	0.73	0.00	0.94	0.67
relative welfare	1.00	1.00	1.03	1.14	1.07
standard deviation c^e	0.11	0.11	0.09	0.07	0.10
standard deviation c^u	0.072	0.071	0.056	n/a	0.063

Notes: Default costs affect only employment-to-unemployment (E2U) transitions, unemployment-to-employment (U2E) transitions or output. Higher match efficiency μ lowers default costs. Relative welfare is computed as a relative consumption equivalent.

eter values the government optimally wants to default at all debt levels, generating in equilibrium an inability to borrow and zero debt levels. Evidently, the U2E cost by itself lacks the asymmetry property of the employment cost of default. Without the possibility of firm exit, default may still increase the interest rate and lower vacancy postings, but the sovereign may prefer to default in times of high (not low) employment. The reason is that depressed vacancy postings are less costly when only a few workers are unemployed. Without the fear of destroying more jobs in times of high employment, there is no extra incentive for the sovereign to refrain from defaulting in such times. This starts an unraveling process, where the government ends up lacking the ability to borrow even when the economy is strong. This alternative setup also exhibits extremely high interest rates whose correlation with unemployment is negative, instead of positive as in the data. But since these high interest rates are inconsequential for firms and workers that are already matched, they do not hurt the economy much. In fact, disregarding endogenous separation attains 3% higher welfare despite de facto exclusion from international capital markets. Qualitatively and quantitatively, it is thus very important in this model not only that new firms face debt limitations, but also that existing firms are financially constrained, in keeping with the vast literature emphasizing the role of working capital constraints (e.g., Christiano and Eichenbaum (1992); Neumeyer and Perri (2005); Jermann and Quadrini (2012); Mendoza and Yue (2012)).

Next, we abstract from an explicit modeling of the labor market altogether and let default directly affect production instead. In the last column, "output," we assume full employment while retaining the disruption from default and default risk on aggregate output. While these punishments cause the same amount of immediate output loss as in the earlier arrangement, they only last for one period. Table 5 shows that this change results in an average spread that is an order of magnitude smaller, and default probabilities shrink. By construction, the model can no longer speak to unemployment variables such as the job-finding rate. In addition, it also fails to generate a positive correlation between private and sovereign interest rates. However, since the economy starts with full production possibilities at the beginning of each period, welfare is considerably higher in this setup.

Finally, we conduct an exercise where we only change the persistence of unemployment in the benchmark model by doubling the parameter of match efficiency μ. We find that the debt-to-GDP ratio sustained over the ergodic set goes down by about 8%, and sovereign spreads fall (column "high μ" in Table 5). When labor market frictions are low, default still triggers a rise in the unemployment rate, but the labor market can recover almost fully in the following period. In contrast, in highly frictional labor markets a one-time disturbance drives up unemployment, and naturally generates higher costs of default because reestablishing jobs takes time.

This gives important insights into one of the main quantitative challenges in the default literature: how to generate substantial default frequencies at the same time as realistic levels of indebtedness. The difficulty of generating frequent default and high indebtedness simultaneously lies in the fact that higher default costs may lead to higher sustainable debt levels but also deter the sovereign from defaulting. Labor frictions can disentangle the two. Evidently, the higher persistence of unemployment in our baseline model creates an additional disincentive to default, which helps create higher debt ratios in equilibrium. Intuitively, since higher persistence prolongs and extends the cost of default, it deters the government from defaulting at similar debt levels and allows it to borrow more at stable default frequencies. At the same time, the higher persistence of unemployment shifts the economy into employment states that incentivize more default due to asymmetry in default costs. Thus the persistence achieves simultaneously higher average spreads. It is thus the interaction between the persistence and the asymmetry of the employment cost of default that explains how our benchmark model can address this challenge.

To conclude, we show that the employment cost of default naturally exhibits asymmetry and persistence, which are key features in quantitative models of sovereign default. Endogenous firm exit in the aftermath of default is particularly important for the workings of the model, while restricting default to hinder firm entry alone appears insufficient to sustain debt in equilibrium.

5.2 Quantifying the cost of default risk

5.2.1 Capturing the Portuguese debt crisis

We use simulation techniques to test whether the model can replicate the dynamics of labor market variables for the Portuguese debt crisis. This serves two purposes: on the one hand, the exercise helps evaluate the fit of the model beyond the moments targeted in the estimation, and on the other hand, it increases confidence in the subsequent quantification of default risk. To carry out the simulation, we draw random productivity shocks for $200,000$ periods, simulate the model forward and discard a burn-in period, starting the simulations from mean productivity and employment levels and zero debt. We then focus on *debt crises*, which we define as episodes where annualized spread exceeds 5% in one period, and which are neither preceded nor succeeded by default in the four-year window around this quarter. Figure 5 illustrates such instances where sovereign spreads rise substantially but do not lead to default, plotting the quarter of these elevated spreads at time 0. On top of the simulated model data, we plot the actual time series from Portugal between 2010 and 2014 (red solid lines).

Figure 5 reveals that the model performs well in capturing the movements of key labor market variables during the Portuguese debt crisis. At simulated sovereign interest spreads that are similar to their empirical counterparts, unemployment rates increase both in the model and in the data. The median model-generated spread increases to the same level and at the same time as in the data. The unemployment rate peaks one quarter after the end of the crisis period, a bit earlier than in the data, where unemployment continues to rise. However, despite exhibiting slightly more persistence in unemployment, the empirical time series lies within the 25th and 75th quantiles of the model simulations, fostering confidence in the ability of the model to capture the data. The model also performs well in matching the higher probability of separation during

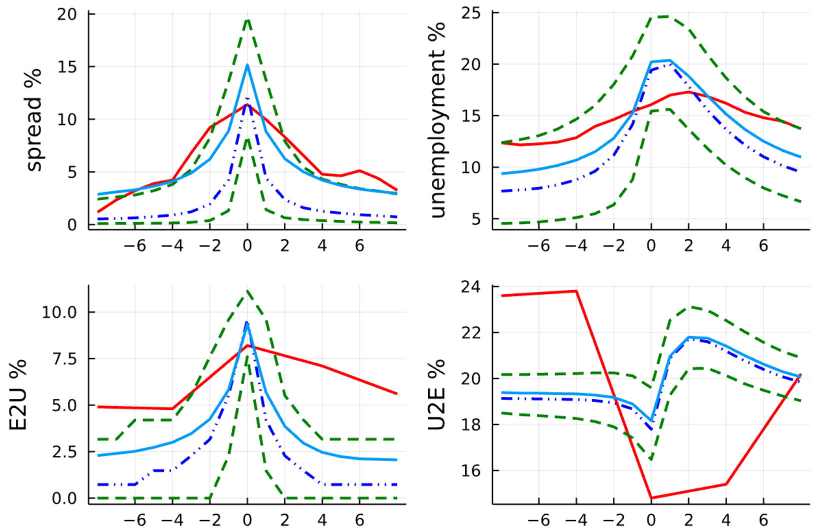

Figure 5: Debt crisis; simulated model mean (solid blue), median (dashdot blue) and quartiles (dash green) versus data (solid red) over time.

the debt crisis with an E2U transition rate that rises to about 8% both in the model and data. Finally, while job-finding rates slow in the run-up to the peak of the crisis in accordance with the data, U2E rates show a faster recovery in the model, contributing to slightly lower persistence in unemployment.

To understand what drives labor market dynamics in the model, it is useful to study the development of other economic indicators during debt crises. Figure 6 shows that debt crises typically occur as a result of a long and moderate fall in productivity. During the slow decline in productivity, the value of providing insurance outweighs the cost of default, and the government ramps up the debt-to-GDP ratio at an increasingly high risk premium. This translates into higher financing costs for firms. In response, firms adjust to higher loan rates R and elevated default risk with lower vacancy postings and more layoffs, prompting labor market tightness θ to decrease. Notably, firms cut vacancy postings when they anticipate default not only because job vacancies become more expensive but also because the expected job separation rate rises. With a static labor market, the latter effect would be absent. Output simulations show a significant contraction during this period, in keeping with Portugal's GDP loss, and reflect both the decline in productivity and the simultaneous drop in employment. Despite driving up the debt ratio, the government fails to prevent a substantial drop in welfare over the period. The typical debt crisis ends when the productivity drop reverses and the economy faces a strong recovery of U2E transitions, labor market tightness and output,

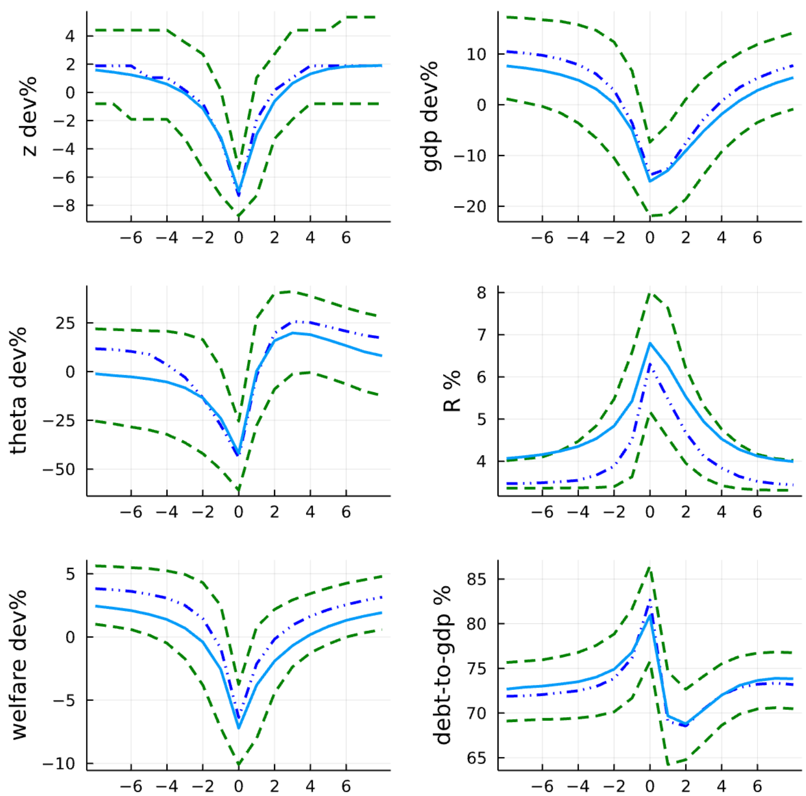

Figure 6: Debt crisis; simulated model mean (solid blue), median (dashdot blue) and quartiles (dash green) over time.

allowing the government to lower its indebtedness.

This exercise shows that the model does well at replicating the fact that default *risk* alone has a profound impact on economic activity. Our model is therefore distinct from theories where only the default decision itself triggers endogenous or exogenous changes in the real economy. The reason is that the model in this paper features anticipation effects of default that hinge on the dynamic nature of the labor market and the effect of spreads on private lending. Our model can thus capture the labor market movements in the Portuguese debt crisis in the early 2010s, despite Portugal's ultimate non-default.

5.2.2 How costly is anticipation of default?

As a next step, we analyze the extent to which the sovereign debt crisis affected the working population. In other words, we seek to quantify how costly default risk or the mere anticipation of a potential default is on workers. But determining the degree to which default risk contributes to poor economic conditions and the degree to which it

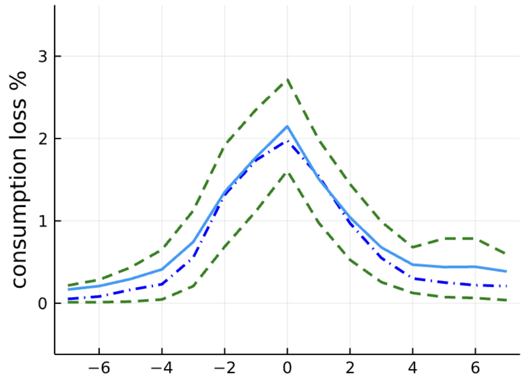

Figure 7: Debt crisis; consumption loss of anticipating default.

results from those is challenging. For instance, the alternative scenario without anticipated default is likely to feature a recession because (exogenous) productivity slowed during the same time with negative consequences for the labor market. Moreover, when abstracting from default risk, the endogenous borrowing constraint vanishes, leaving an impatient government with the incentive to increase its bond issuance indefinitely.

To address these challenges, we start by filtering out the simulation episodes from above that best match the spreads in the data for Portugal. Specifically, for each simulated path of interest rate spread we compute its distance to the actual spread time series and attach a proportional probability weight to it. This effectively computes a posterior probability of the simulated series. We then take the 5% of all paths that are closest in mean square error to the one observed in the data and filter out the corresponding productivity paths. These paths will stand in for the exogenous component of Portugal's debt crisis in our further analysis and will be taken as given in our counterfactuals.

To isolate the cost effect of rising default probabilities from other fundamental changes, we shut down feedback from lower bond prices q on firm's lending conditions R. This breaks the link from potential default fears on the part of lenders to the labor market because it effectively forces creditors to assume zero default risk. We retain the government's initial debt policy but ensure that the fiscal budget constraint is satisfied through the readjustment of taxes and benefits. The counterfactual thus describes the case where the government issues the same amount of debt but at prices that do not reflect any default risk, allowing for some previously lost revenue to flow back to workers.

The result of this exercise is summarized in Figure 7, which plots an employed worker's consumption loss from default risk over the Portuguese debt crisis. As default becomes more likely, the average employed worker starts facing a quarterly consumption loss that peaks at 2.1% at the height of the crisis. Of course, this loss adds to the consumption drop caused by the underlying recession over this period. It also adds to the fact that an employed worker may face a higher probability of job loss and in that case faces less likelihood of finding a new job. This consumption loss is thus just the part of the anticipation costs that can solely be attributed to default risk given that the worker keeps her job. In addition, some default risk may still be transmitted to the labor market in the counterfactual because the level of sovereign debt issuance B' keeps affecting private sector interest rate R. If one believes that some of this comovement reflects default risk, we underestimate the size of the consumption loss. This number is thus likely to reflect the lower bound of the welfare loss incurred by anticipating default.

The occurrence of such anticipatory welfare losses poses a series of further questions: First of all, anticipatory welfare losses may produce an argument for avoiding default risk altogether, for example by changing debt policy. However, the exercise is based on the government's optimal choices, and so avoiding default risk through a different policy mix without changing anything else cannot be optimal. Second, one may wonder how close Portugal was to default, and what impact default would have had on consumption. In other words, what are the welfare consequences of deciding to default in a debt crisis similar to Portugal's? Finally, what other measures could public policy utilize in order to address costs for workers during crises, especially by directly targeting the labor market? We turn to these questions next.

6 Counterfactual policies

6.1 Default

One common topic of debate among policy makers relates to the desirability of defaulting earlier in time in the hope of achieving better economic conditions sooner. Why do troubled countries often delay a seemingly inevitable default? Couldn't an earlier default secure lower spreads sooner and benefit the economy, even at a short-run cost? These are open questions that are relevant to the borrowing crises faced by many countries. Here, we start investigating the costs and benefits from a counterfactual default on the Portuguese debt obligations in 2012.

To design the counterfactual scenario, we reuse the same productivity sequence that informed the Portuguese debt crisis, but force the government to default at time 0 when

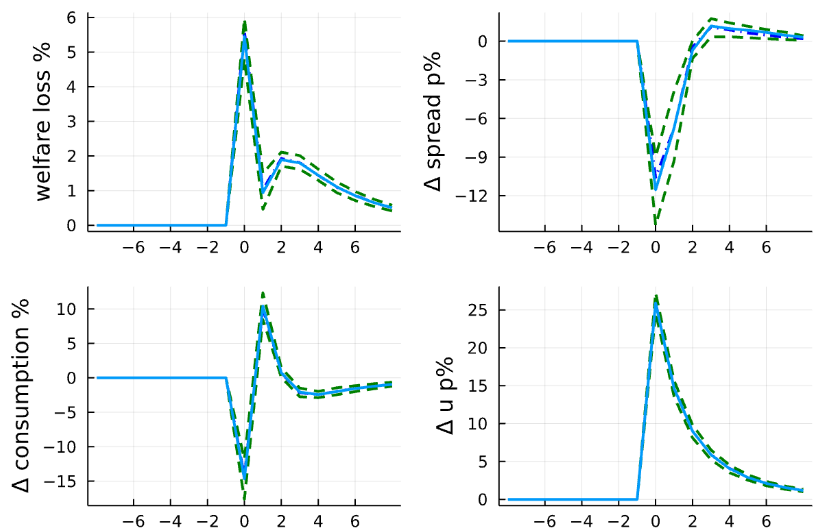

Figure 8: Counterfactual default; percent (%) and percentage point (p%) changes.

spreads peaked. After hypothetically defaulting, the government fully reoptimizes its policy. We thus impose the condition that the simulation coincide with the debt crisis from Section 5.2.1 before the default event, but resimulate the model afterward.

Since the default decision is forced upon the government, while it abstained from it in the original simulation, the counterfactual economy naturally suffers a welfare loss (about 6%) at time 0 in this experiment. More interestingly, as Figure 8 shows, the economy continues experiencing lower welfare in the resolution of the crisis at each point in time. This circumstance thus indicates that defaulting early does not come at a release ex-post as hoped for by those advocating for earlier defaults, at least in Portugal's case. This is somewhat surprising, because one may assume that an economy without debt may be able to borrow cheaply again, benefiting from large capital inflows that may boost welfare in the aftermath of a default. To investigate what factors explain the continued welfare loss, Figure 8 plots the changes in further economic indicators. We find that there are indeed large capital inflows into the economy, and thus an uptick in consumption immediately after default. However, this consumption stimulation only lasts for one quarter, and then consumption falls below the baseline crisis level again. It turns out that the unemployment rate, after shooting up in the default quarter, keeps putting a persistent strain on the economy. Aggregate GDP mirrors the persistent unemployment increase. Even sovereign spreads indicate worse conditions compared to the debt crisis without default, because the economy has entered a critical employment state after the initial default that it cannot outgrow quickly enough. Overall, the "clean slate" from debt defaulting is not sufficient to turn around the adverse welfare consequences of default, even when taking the default quarter out of the picture.

6.2 Labor policy

The preceding analysis has shown that debt and default crises occur at the same time as unemployment levels rise. Shifting patterns in worker mobility further aggravate these adverse conditions, which are costly for workers. This section then investigates whether the government can counteract some of the negative consequences of default risk by implementing policies that directly aim at supporting the labor market. One may consider initiatives that ease the job search for the unemployed through creation of job agencies. Alternatively, one could imagine the implementation of job training programs, agencies that support the formulation and distribution of applications, etc. In fact, a vast number of labor market policies are conceivable. To make the analysis manageable, we thus take the approach of Murtin and Robin (2018), arguing that most active labor market policies are likely to affect only a limited number of structural parameters in the model: match efficiency, the job destruction rate and the cost of vacancies. In addition, we investigate how unemployment benefits affect the cost of default risk on workers.

The implications of such policies may not be straightforward because they are likely to affect a government's ability to commit to debt repayment. Therefore, we proceed in two steps. First, we allow labor market policies to take effect and shift the private sector equilibrium, but keep sovereign debt issuance and default policies constant. This isolates the direct potential effect the policies have on the economy. Second, we resolve the model for each policy change to account for general equilibrium effects. Those include how public labor policies interact with the optimal debt policy and allow us to investigate how this interaction affects default frequencies and debt ratios.

The results are summarized in Table 6, which lists key economic indicators for the baseline model next to the direct and total effects of four different labor policies. The first describes a 20% increase in replacement ratio ζ for unemployed workers. Columns a show policies that cut vacancy posting costs in half. The match efficiency parameter μ is doubled in a third exercise. Lastly, we halve the exogenous separation parameter ξ. One common characteristic of these policies is that while direct effects show quite substantive deviations from the baseline in terms of average spread and default frequencies, these

Table 6: Counterfactual labor policy.

	baseline	direct effect				total effect			
		ζ	a	μ	ξ	ζ	a	μ	ξ
A. unconditionally									
average spread (%)	2.2	2.5	1.7	0.9	0.7	2.2	2.2	1.8	1.1
unemployment rate	0.09	0.09	0.07	0.04	0.05	0.09	0.07	0.04	0.05
job-finding rate	0.21	0.19	0.23	0.41	0.15	0.21	0.28	0.70	0.19
debt-to-GDP ratio	0.73	0.73	0.72	0.70	0.71	0.72	0.71	0.67	1.16
relative welfare	1.00	1.05	1.06	1.06	1.06	1.01	1.03	1.07	1.07
B. crisis peak									
average spread (%)	15	15	13	11	6.8	14	14	11	8.9
unemployment rate	0.20	0.20	0.19	0.15	0.11	0.20	0.18	0.14	0.12
consumption cost (%)	2.1	2.1	2.0	1.6	1.0	2.0	2.1	2.2	2.6

Notes: Direct effect with constant debt-default policy; total effect with general equilibrium effects; policies affect benefits (ζ), vacancy posting costs (a), match efficiency μ and exogenous separation ξ.

changes are substantially smaller when we allow the government to readjust its debt and default policy. For example, doubling match efficiency μ lowers average spreads from 2.2% to 0.9% without further policy adjustments, but most of this reduction is undone when looking at the total effect (1.8%). Similarly, we find that labor market policies have the potential to considerably lower the consumption cost of default risk during debt crises, but under general equilibrium readjustments they increase or even overshoot the baseline cost. For instance, lowering ξ has a direct effect of reducing the consumption cost of default from 2.1% to 1.0%, but the total effect shows an increase to 2.6%. Table 6 also documents the fact that when a government reoptimizes its debt and default policy, the effects on average unemployment appear to stay fairly constant, while job-finding rates tend to rise. This happens most notably under the match efficiency policy μ that boosts the job-finding probability to 0.41 without debt policy reoptimization, and all the way to 0.70 with it. At the same time, allowing the government to readjust does not necessarily increase welfare. Benefit and vacancy cost policies generate larger direct welfare gains if public debt policy is kept constant, indicating that commitment issues are costly here. Finally, an increase in the replacement ratio for unemployed workers has only moderate direct and total effects. In summary, our findings indicate that benefit policies seem relatively ineffective at cushioning workers against the cost of default risk, and that allowing for general equilibrium effects can have important impacts on the effectiveness of labor market policies.

7 Conclusion

This paper presents an equilibrium default framework of sovereign debt with search frictions, in which the government not only faces a trade-off between taxation and the financial and labor market disruptions caused by defaulting, but is also concerned with mitigating unemployment well in advance of an actual default. The endogenous employment cost of default arises because public borrowing risk transmits to workers, who face worse labor market conditions.

The model is estimated to match the Portuguese debt crisis in the early 2010s and captures the empirical employment pattern well. Persistent unemployment and asymmetric default costs ensure that a government can sustain realistic debt-to-GDP ratios while default frequency is kept at reasonable levels. The main quantitative results indicate that sovereign risk causes significant consumption losses for workers. Unemployment and worker mobility worsen when borrowing risk culminates in default. Labor market policies can only moderately offset these costs.

While the results indicate that for Portugal, defaulting would have come at persistent welfare costs, it would be interesting to apply an anticipatory framework to a wider range of countries to study alternative timings of defaults and to quantitatively weigh short-term vs medium-run costs in debt crises. Introducing long-term bonds, additional costs from market exclusion, or debt haircuts are natural model extensions that we leave for further research. Another interesting avenue would be to include habit formation and other mechanisms that increase the persistence of model variables. Furthermore, the novel source of multiplicity suggests a possibility to study fundamental- and expectation-driven default crises in a unified framework. We thus hope that the proposed theory and tractability approach may prove useful for further research.

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A Appendix: Microfounding $\mathcal{R}(\cdot)$

This section provides a microfoundation for the reduced-form rule in equation (7) that links a firm's lending rate R_t to a series of economic indicators. The proposed setup is based on the crucial role intermediaries play in financial economies and follows the work of Gertler and Karadi (2011).

We incorporate financial intermediation by refining lenders' preferences. To do so, we assume that foreign lenders are made up of a continuum of identical financial households. Each financial household consists of members that switch between acting as either depositors or bankers. Depositors make consumption-saving decisions on behalf of households while bankers provide loans to firms.

Specifically, depositors choose consumption c_t^f and within-period deposits x_t, and buy government bonds B_{t+1}^f at price q_t. They face the following optimization problem

$$\max \ \mathbb{E}_t \sum_{t=0}^{\infty} \left(\tfrac{1}{1+r}\right)^t c_t^f$$

$$\text{s.t.} \quad c_t^f = d_t \left(B_t^f - q_t B_{t+1}^f \right) + R_t^x x_t - x_t + \pi_t,$$

where R_t^x is the non-contingent gross return on deposits and π_t are the bankers' net earnings. Note that holding government bonds is risky because the government may not repay ($d_t = 0$), and bonds B_{t+1}^f cannot exceed total aggregate bond issuance $B_{t+1}^f \leq B_{t+1}$. Risk neutrality implies that public debt is still priced at $q_t = \mathbb{E}_t \left(\frac{d_{t+1}}{1+r} \right)$, which follows from the first order condition of the depositor's maximization problem with respect to bond holdings. Since depositors are happy to hold any given share $\iota \in [0,1]$ of aggregate bonds, we let $B_t^f = B_t$ in equilibrium. Alternatively, one could also assume that only a subset of lenders consists of financial households and then get $B_t^f = \iota B_t$. Furthermore, the first order condition of depositors implies $R_t^x = 1$.

Bankers, who start the period with net worth a_t, obtain deposits x_t from other households and issue firm loans l_t, thus facing the balance sheet:

$$l_t = a_t + x_t.$$

Equity growth crucially depends on the difference between the return on firm loans $1+R_t$ and the interest rate on deposits R_t^x:

$$a_{t+1} = (1 + R_t) l_t - R_t^x x_t = R_t l_t + a_t.$$

Using the fact that $R_t^x = 1$, the bankers' problem can be easily characterized if there exists an arbitrage opportunity due to market imperfections that renders loans profitable,

$R_t \geq 0$. In this case it is optimal for bankers to build up net worth indefinitely. To restrict this, we introduce a common incentive constraint that limits bankers' ability to borrow. Furthermore, whenever bankers switch roles with depositors within a household, the new start-up equity is limited to a constant α plus bond holdings such that π_t are dividends and retained earnings from old bankers less the endowment of new bankers. A banker's objective is therefore to maximize

$$\begin{aligned} \mathcal{P}_\tau = \max \quad & \mathbb{E}_\tau \sum_{t=0}^{\infty} (1-\lambda)\lambda^t \left(\tfrac{1}{1+r}\right)^t a_{t+1} \\ \text{s.t.} \quad & a_{t+1} = R_t l_t + a_t \\ & \mathcal{P}_\tau = \gamma l_t, \end{aligned}$$

where λ is the probability of retaining the current role as banker and γ governs the strength of the incentive problem. The incentive constraint can be interpreted as a standard enforcement problem between the banker and her depositors. The idea is that in each period, bankers can choose to divert a share γ of their assets, in which case depositors can recover only the remaining share $(1-\gamma)$.

Second, the value of a banker $\mathcal{P}$ and her loan supply l_t are linear in her net worth. To see this, note that the problem can be reformulated as

$$\begin{aligned} \mathcal{P}_t =& \nu_t l_t + \eta_t a_t \\ \nu_t =& \mathbb{E}_t \{(1-\lambda)(1 + R_t - R_t^x) + \lambda \left(\tfrac{1}{1+r}\right) \mathtt{x}_{t,t+1} \nu_{t+1}\} \\ \eta_t =& \mathbb{E}_t \{(1-\lambda) R_t^x + \lambda \left(\tfrac{1}{1+r}\right) \mathtt{z}_{t,t+1} \eta_{t+1}\}, \end{aligned}$$

where $\mathtt{x}_{t,t+1} = \frac{l_{t+1}}{l_t}$ is the gross growth rate in assets and $\mathtt{z}_{t,t+1} = \frac{a_{t+1}}{a_t}$ is the gross growth rate in net worth. Similarly, the incentive constraint can be expressed as a linear function of net worth:

$$l_t \leq \frac{\eta_t}{\gamma - \nu_t} a_t = \chi_t a_t.$$

Notably, the leverage ratio χ_t – the ratio between assets and equity – depends on R_t and is time-varying. However, since χ_t does not depend on anything specific to a particular banker, aggregate loan supply L_t^s can be expressed as a function of aggregate wealth A_t, which in turn depends on q_t and B_{t+1}:

$$\begin{aligned} L_t^s =& \chi_t A_t \\ A_t =& \lambda \left[1 + R_{t-1} \chi_{t-1}\right] A_{t-1} + \alpha + \iota q_t B_{t+1}. \end{aligned}$$

On the demand side, the financing needs of share k_{jt} of output by firm j translate into an aggregate loan demand equal to $L_t^d = (1-\xi(z_t))(1-s_t)(N_t+m_t)k_t z_t + av_t$. The loan market clears when the price of loans R_t equalizes supply and demand, i.e.,

$$\chi_t A_t = (1-\xi(z_t))(1-s_t)(N_t+m_t)k_t z_t + av_t.$$

Since χ_t, s_t, v_t and k_t are all functions of R_t, A_t is a function of q_t and B_{t+1} (and potentially further variables []), and the loan equilibrium clearly depends on z_t and N_t as well, this implicitly pins down R_t, $f(R_t, q_t, B_{t+1}, z_t, N_t, []) = 0$. Hence, the loan rate will be a function with (at least) these arguments, $R_t = \mathcal{R}(z_t, N_t, q_t, B_{t+1})$.

B Appendix: Proofs

Proof of Lemma 1. We sum over all wages in a given period:

$$\begin{aligned}
w &= N(1-\xi(z))\int_i (1-s_{j(i)})w^e_{j(i)}di + m(1-\xi(z))\int_i (1-s_{j(i)})w^u_{j(i)}di && (15)\\
&= N(1-\xi(z))\,\mathbb{E}[(1-s_{j(i)})z(1-k_{j(i)}R)] && (16)\\
&= N(1-\xi(z))P(s_{j(i)}=0)\,\mathbb{E}[z(1-k_{j(i)}R)|k_{j(i)}\ s.t.\ s_{j(i)}=0] && (17)\\
&= N(1-\xi(z))(1-s)z(1-kR) && (18)\\
&= \underbrace{N(1-\xi(z))\frac{\phi}{\mathcal{R}(z,N,q,B')}}_{\text{number of previously employed}} \quad \underbrace{z(1-\phi)}_{\text{average wage of previously employed}}, && (19)
\end{aligned}$$

where $w^e_{j(i)}$ is wage of worker i who was previously employed at firm j, and $w^u_{j(i)}$ is wage of worker i who was previously unemployed and now matched with firm j. □

Proof of Lemma 2. Rewrite equation (12) with $N'{=}1{-}u{=}\mathcal{N}(\Omega;\delta,\mathcal{D})$, $\lambda^w{=}\Lambda^w(\Omega;\delta,\mathcal{D})$:

$$\begin{aligned}
\mathcal{W}(\Omega;\mathcal{D}) &= \max_\delta N\mathcal{E}(\Omega,\delta;\mathcal{D}) + (1-N)\mathcal{U}(\Omega,\delta;\mathcal{D})\\
&= \max_\delta [N + (1-N)\lambda^w](1-\xi(z))(1-s)[\mathbf{u}(c^e) + \beta\,\mathbb{E}\,\mathcal{E}(\Omega',\mathcal{D};\mathcal{D})] +\\
&\quad [[N+(1-N)\lambda^w](1-(1-\xi(z))(1-s)) + (1-N)(1-\lambda^w)][\mathbf{u}(b) + \beta\,\mathbb{E}\,\mathcal{U}(\Omega',\mathcal{D};\mathcal{D})]\\
&= \max_\delta (1-u)\mathbf{u}(c^e) + u\mathbf{u}(b) + (1-u)\beta\,\mathbb{E}[\mathcal{E}(\Omega',\mathcal{D};\mathcal{D})] + u\beta\,\mathbb{E}[\mathcal{U}(\Omega',\mathcal{D};\mathcal{D})]\\
&= \max_\delta (1-u)\mathbf{u}(c^e) + u\mathbf{u}(b) + \beta\,\mathbb{E}[N'\mathcal{E}(\Omega',\mathcal{D};\mathcal{D}) + (1-N')\mathcal{U}(\Omega',\mathcal{D};\mathcal{D})]\\
&= \max_\delta \mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{W}(\Omega';\mathcal{D})].
\end{aligned}$$

□

Proof of Lemma 3. We show this by means of Blackwell's sufficient conditions.

First, to show monotonicity, assume that $\mathcal{W}(\Omega;\mathcal{D}) \leq \mathcal{Y}(\Omega;\mathcal{D})\,\forall\,\Omega$ with Bellman operator T and suppose $\delta^\star$ is optimal policy for continuation value function $\mathcal{W}$. Then:

$$
\begin{aligned}
(T\mathcal{W})(\Omega;\mathcal{D}) = &\quad \sup_{\delta} \quad \mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{W}(\Omega';\mathcal{D})] \\
&= \mathcal{N}(\Omega;\delta^\star,\mathcal{D})\mathbf{u}(c^{e\star}) + (1-\mathcal{N}(\Omega;\delta^\star,\mathcal{D}))\mathbf{u}(b^\star) + \beta\,\mathbb{E}[\mathcal{W}(z',B'^\star,N'^\star;\mathcal{D})] \\
&\leq \mathcal{N}(\Omega;\delta^\star,\mathcal{D})\mathbf{u}(c^{e\star}) + (1-\mathcal{N}(\Omega;\delta^\star,\mathcal{D}))\mathbf{u}(b^\star) + \beta\,\mathbb{E}[\mathcal{Y}(z',B'^\star,N'^\star;\mathcal{D})] \\
&\leq \sup_{\delta}\mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{Y}(z',B',N';\mathcal{D})] \\
&= (T\mathcal{Y})(\Omega;\mathcal{D}).
\end{aligned}
$$

Note that ω and $\mathcal{N}$ depend on the continuation value only through $\mathcal{Q}$, see Lemma 1. Since we focus on a given bond price schedule $\mathcal{Q}$ that satisfies the equilibrium selection criterion, δ^* stays feasible under $\mathcal{Y}$.

Second, to show discounting, note that adding a constant Δ to an optimization problem does not affect optimal choice:

$$
\begin{aligned}
(T\mathcal{W})(\Omega;\mathcal{D}) = &\quad \sup_{\delta} \quad \mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{W}(\Omega';\mathcal{D}) + \Delta] \\
= &\quad \sup_{\delta} \quad \mathcal{N}(\Omega;\delta,\mathcal{D})\mathbf{u}(c^e) + (1-\mathcal{N}(\Omega;\delta,\mathcal{D}))\mathbf{u}(b) + \beta\,\mathbb{E}[\mathcal{W}(\Omega';\mathcal{D})] + \beta\Delta \\
&= (T\mathcal{W})(\Omega;\mathcal{D}) + \beta\Delta
\end{aligned}
$$

Monotonicity and discounting then establishes the Bellman operator T is a contraction, see Theorem 3.3 in Stokey, Lucas, and Prescott (1989). □