Expectation-driven Bond Pricing with Dynamic Private Sector Behavior*

Neele L. Balke†

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Abstract

This paper outlines a novel source for multiple equilibria in sovereign default models. In the model, multiplicity arises due to the presence of employment as an additional and privately determined state variable. If unemployment impacts on the default decision and its evolution depends on sovereign borrowing costs, an economy becomes prone to expectation-driven crises, rendering sovereign debt markets vulnerable to market panics. Pessimistic investors' expectations about default hike sovereign borrowing costs, which translates into higher unemployment and increases the likelihood of a sovereign default, validating the investors' adverse expectations. Three-state multiplicity emerges with neither changing the standard first-mover advantage of the government in the game with investors nor its limited commitment and thus is useful to study fundamental and expectation-driven default crises in a unified framework. The paper shows that policy makers may be able to break bad expectations and increase welfare with repayment guarantees from supranational agencies or fixed floors on debt prices.

Keywords: Sovereign Default, Multiplicity

JEL: E24, E44, E62, F34, F41

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†Department of Economics, University of Chicago, email: nbalke@uchicago.edu, web: http://www.neelebalke.com.

1 Introduction

Multiple equilibria have gained a lot of attention as explanation for the recent sovereign debt crisis in Europe. Policy makers and scientists share the view that besides bad fundamentals also pessimistic expectations can render sovereign debt markets vulnerable to market panics. If this sentiment leads to abrupt movements in bond yields, which retrench the sustainability of government debt and increase the likelihood of a sovereign default, investor fears become self-fulfilling. The market is caught in a bad equilibrium, in which investor's pessimistic expectations about future government default hike sovereign borrowing costs, thus increase the government's incentives to default and validate the perceived riskiness in the first place.

The events surrounding Draghi's speech in July 2012 are widely seen as prime example that self-fulfilling expectations play a role in debt crises: During the European debt crisis from 2010-2012, the sovereign bond spreads of Italy and Spain rose sharply. However, when Draghi announced that the ECB was "ready to do whatever it takes" to tackle the European crisis and introduced the Outright Monetary Transactions (OMT) program, default premia receded and bond spreads fell dramatically although the ECB never actually intervened. A common interpretation of this incidence is that Draghi's speech successfully broke pessimistic expectations and shifted the Eurozone sovereign debt markets out of an adverse equilibrium. In addition, the observed discrepancy between the volatilities of sovereign spreads and economic fundamentals reveals that default premia appear too weakly related to fundamentals and also supports the view that expectations can impact on debt prices.

This paper presents a theoretical model of sovereign debt to understand how expectations about default incentives become self-fulfilling. The model can explain how high interest rates can be caused by adverse market sentiments in addition to poor fundamentals, and why a policy that aims at shifting expectations in an economy trapped in a bad equilibrium can be justified.

The environment follows the workhorse setup in the tradition of Eaton and Gersovitz (1981), which inspired a growing number of quantitative studies.[1] In the benchmark infinite-horizon models following Arellano (2008) and Aguiar and Gopinath (2006), the borrower issues one-period debt at maturity before creditors set bond prices, specifying an interest rate for each possible level of debt issuance: the interest rate schedule. Although these models may intuitively exhibit multiple equilibria – low bond prices limit the ability of the government to raise revenue by debt issuance and so increase default probabilities, justifying low debt prices – the uniqueness result by Auclert and Rognlie (2014) settled this debate. The authors show that for the most widely used variations of the model, there is a unique Markov-perfect equilibrium. Key for this result is the timing assumption that the borrower issues debt before investors set prices, as pointed out by Ayres, Navarro, Nicolini, and Teles (2015) and Lorenzoni and Werning (2014), because a government who understands how the subsequent price schedule depends on its debt decision chooses the unique optimal debt level and corresponding interest rate. The first-mover advantage therefore allows the sovereign to pick the equilibrium with low interest rates.

This model departs from the standard framework by introducing a third and privately determined state, while the first-mover advantage of the sovereign remains unaffected. This minor change is able to break the uniqueness results by producing a novel source of multiplicity and expectation-driven movements in debt prices. Two features of the state are important in order for multiplicity to emerge: First, it impacts on the default decision. Second, its dynamic evolution depends on the government's default probability. The additional state variable that gives rise to the three-state multiplicity in this paper is employment. On the one hand, higher unemployment increases the likelihood of default because the servicing of debt becomes more difficult when it dampens production. Furthermore, the disincentive to default manifests itself in terms of employment cost with the required property that the default punishment decreases in the unemployment rate (Balke (2017)). On the other hand, worse borrowing conditions of the government foster higher unemployment – a plausible assumption given

[1]Starting with Aguiar and Gopinath (2006) and Arellano (2008), many studies have analyzed various aspects of sovereign debt crises, for an overview see Aguiar, Chatterjee, Cole, and Stangebye (2016).

the dependence of firms on private borrowing and the co-movement of sovereign and private spreads.[2]

The intuition behind the three-state multiplicity is that in a model with risky debt, lenders' pessimistic expectations about the likelihood of default drive up risk premia on debt, which translates into higher unemployment and encourages default, justifying the initial pessimism. If investors fear that future unemployment rates will be high and therefore attach a high default probability to government borrowing, the debt price is pinned down by a bad debt schedule. This makes it relatively costly for the government to borrow compared to a situation where optimistic investors coordinate on a good price schedule. However, adverse default expectations become self-fulfilling because a bad debt price and limited debt issuance imply high unemployment, making default more appealing.

These belief-driven debt crises are not a mere theoretical curiosity. In fact, recent work by Bocola and Dovis (2016) shows that risk of self-fulfilling crises contributed non-negligibly to the movements in Italian bond yields. Therefore, it seems crucial to consider the possibility of non-fundamental crises when designing macroeconomic policies. This paper finds that policy makers, who want to react to the risk of being caught in a bad market equilibrium with low welfare, may be able to successfully break bad expectations with repayment guarantees from supranational agencies or fixed floors on debt prices.

2 Related literature

This work is related to the literature on the existence and uniqueness or multiplicity of equilibrium in sovereign default models. For a model with temporary or permanent exclusion from markets as default punishment, Chatterjee and Eyigungor (2012) prove the existence of a downwards-sloping equilibrium price function for long-term debt under the condition that the persistent and discrete endowment is complemented by

[2]These properties are explicitly modeled in Balke (2017) and in line with empirical findings (Acharya, Eisert, Eufinger, and Hirsch (2015)).

an iid component with continuous support. However, to date it has been very hard to prove existence of an equilibrium when endowments and debt levels are continuous and it is an open question of how the possibility of reentering financial markets in contrast to a permanent exclusion may alter existence results in the standard setup.

Auclert and Rognlie (2014) establish uniqueness of equilibrium in the benchmark infinite-horizon debt model of Eaton and Gersovitz (1981) with one-period debt, an endowment state that follows an exogenous Markov process and with a default punishment that involves permanent autarky. If the value from sovereign default is exogenous through permanent market exclusion, their mimicking-at-a-distance proof shows that the model's unique Markov-perfect equilibrium is also its unique subgame-perfect equilibrium. They further extend their uniqueness result to two modifications of the standard framework. In the first variation, the government is able to save positive amounts of debt before default. The second alternative considers the case with an endogenous value of default stemming from the popular assumption that a defaulting government can borrow again after a stochastic number of periods. In the case of market reentry there do not exist two distinct equilibrium price functions of the form where one dominates the other. This paper proposes a model with a third state variable, which breaks the uniqueness result of Auclert and Rognlie (2014).

Several other default models in the literature present alternative ways through which uniqueness of equilibrium can be relaxed. For example, a different timing assumption than in the benchmark model can induce failed auctions. In Cole and Kehoe (2000), the government does not issue new debt at the same time as making its repayment or default decision. Instead, the government sells new debt to international investors and only after observing the outcome of this auction, it decides whether or not to repay the debt carried over from the previous period. Furthermore, defaulting is more likely if the auction unfolds low debt prices because repayment is more costly with less current revenue and enough risk aversion of domestic agents. Then self-fulfilling rollover crises emerge when pessimistic investors set low debt prices, which makes it costly to rollover debt and leads to default and thus verifies their original pessimism. The rollover crisis could be avoided when optimistic investors set good debt prices and induce the government to repay. Thus, the change in the timing assumption introduces a coordination problem among investors that leads to multiplicity because their expectations become self-fulfilling. In fact, the government may expose itself to the risk of this crisis by gambling for redemption (Conesa and Kehoe (2015)). Chatterjee and Eyigungor (2012), Aguiar, Amador, Farhi, and Gopinath (2015) and Aguiar, Chatterjee, Cole, and Stangebye (2017) also generate belief-driven rollover crises based on similar model mechanics. The proposed model differs from these paper because the government cannot default after internalizing the outcome of an initial debt auction and is therefore not vulnerable to a rollover crisis.

Another multiplicity result traces back to Calvo (1988) who models the government as a price taker. In the Calvo (1988) model, the government issues current debt instead of debt at maturity as in Eaton and Gersovitz (1981) and inherits a debt level that includes endogenously determined interest payments in the following period. The interest rate schedule is therefore a function of current debt, not outstanding debt repayments. Since default is a better option if the overall debt burden is high, there exist two possible interest rates that let investors break even: one with low interest rates, low debt repayments and a low default probability and another with high interest rates, high debt burden and a high debt repudiation. Either of those two equilibria can arise from self-fulfilling expectations. This type of multiplicity is sometimes referred to as Laffer curve multiplicity because the bond revenue as a function of debt issuance is inverted U-shaped. In the bad equilibrium, the economy finds itself on the downward-sloping part of the Laffer curve. A Laffer curve is not present in this work.

Lorenzoni and Werning (2014) provide a dynamic version of the Calvo (1988) model, in which a government needs to finance a certain exogenous deficit. The authors contest that governments can commit to an initial level of bond issuance because governments may have the incentive to issue more debt after a failed auction with low debt prices to cover their expenses. Similarly to Calvo (1988), default fears entailing low debt prices are justified if the following higher indebtedness raises default probabilities in the following period, and optimistic expectations can equally be an equilibrium outcome when low interest rates limit new debt issuance.

A similar route is taken by Ayres, Navarro, Nicolini, and Teles (2015) who provide a model in which governments lose their first-mover advantage and are unable to select a point on the good side of the Laffer curve. They show that the crucial element for the uniqueness of equilibrium of the benchmark Eaton and Gersovitz (1981) model is the timing of moves in addition to the maturity structure of debt. In the model studied here, the government chooses debt at maturity, pinning down directly what is owed in the following period, and retains its first-mover advantage, avoiding the bad side of the Laffer curve.

Passadore and Xandri (2015) find multiplicity when the lower bound on debt is zero. This restricts the state base for debt compared to the standard setup. However, Auclert and Rognlie (2014) prove that uniqueness of the benchmark setting continues to hold if the exogenous bound on savings is strictly positive. There is no restriction on borrowing or saving in this model.

Relaxing the one-period structure of sovereign debt, Stangebye (2015), Stangebye (2017) and Aguiar and Amador (2016) show that multiple debt schedules are possible in the presence of long-duration bonds. Long-term debt makes the economy vulnerable to debt dilution problems. Investors with pessimistic sentiments about long-run default incentives demand higher spreads that drive up borrowing which in turn dilutes the value of long-term debt, increases default probabilities and validates initial fears. This paper considers one-period debt as in the standard framework.

To sum up, the existing literature on equilibrium multiplicity in sovereign debt models has focused on changing key assumptions of the standard setup to break its uniqueness result: the timing of moves, the maturity structure of debt, the feasible set of debt issuance or the duration of bonds. In this paper, another state variable is introduced in the workhorse model, which can act as a novel source of multiplicity.

3 Model

This section outlines a simple three-period model to explain the occurrence of multiple equilibria in sovereign debt models stemming from the presence of an additional state variable that is privately determined. It highlights the key components for this three-state multiplicity to break the equilibrium uniqueness of the standard model and puts it in relation to other sources of multiplicity.

3.1 Agents, timing and equilibrium

A small open economy is populated with either employed or unemployed workers, a government who maximizes workers' welfare and investors who buy sovereign bonds. There are three periods indicated by $t = \{1, 2, 3\}$.

Agents. The economy consists of a measure one of workers. In period t, a share N_t of workers is employed and a share $(1 - N_t)$ remains unemployed. Employed workers earn a wage w_t and pay taxes τ_t while the unemployed receive constant unemployment transfers T. Their is no insurance against income fluctuations from changes in their employment status by pooling their income. Workers receive utility $u(c_t)$ from consumption c_t and discount future utility with β. They are risk-averse with constant relative risk aversion such that the utility function is of the CRRA form, $u(c) = \dfrac{c^{1-\sigma} - 1}{1 - \sigma}$. Following the standard assumption in the literature on sovereign default, workers cannot save such that employed agents consume $c_t = w_t - \tau_t$ and unemployed workers achieve the consumption level equal to their benefits, $c_t = T$.

The government wants to maximize a utilitarian social welfare function over the three periods by choosing taxes, default and debt issuance. Its objective is given by:

$$\begin{aligned} \max \mathbb{E}\big\{ & N_1 u(w_1 - \tau_1) + (1 - N_1)u(T) + \beta[N_2 u(w_2 - \tau_2) + (1 - N_2)u(T)] + \\ & \beta^2[N_3 u(w_3 - \tau_3) + (1 - N_3)u(T)]\big\} \end{aligned} \tag{1}$$

In the first period, the government chooses to issue sovereign debt B_2 at price q_1, which matures in period $t = 2$, and taxes workers by setting lump-sum taxes τ_1. In the second period, the government sets taxes τ_2 and chooses to default or repay $d_2 \in \{0, 1\}$ the full amount of maturing debt, where $d_2 = 1$ indicates repayment and $d_2 = 0$ describes a default event. In the third period, the government decides on the tax rate

τ_3. The government cannot commit to future policies in earlier periods.

The government's budget constraints

$$(1 - N_1)T = N_1\tau_1 + q_1 B_2 \tag{2}$$

$$(1 - N_2)T = N_2\tau_2 - d_2 B_2 \tag{3}$$

$$(1 - N_3)T = N_3\tau_3 \tag{4}$$

require that total transfer spendings equal revenues in each period. Besides, the sovereign faces a price schedule that renders the debt price dependent on the total amount of borrowing:

$$q_1 = Q(B_2) \tag{5}$$

Further constraints are imposed by the laws of motion for employment, which is not only influenced by previous employment levels but also by sovereign borrowing, debt prices and the default decision:

$$N_1 \quad \text{given} \tag{6}$$

$$N_2 = H_2(N_1, q_1, B_2) \tag{7}$$

$$N_3 = H_3(N_2, d_2) \tag{8}$$

These laws of motion can be motivated by the fact that firms tend to create more jobs when private credit conditions are good and that there are strong correlations between private and public credit spreads (see also Section 4). Defaults are often associated with banking crises which worsen the lending conditions for firms. The cost of default is implicitly included in (8) through the possibility of default to cause higher unemployment.[3]

Wages in the first and last period, w_1 and w_3, are deterministic while second period

[3]See Balke (2017) for a detailed model of the employment cost of default.

wages w_2 are stochastic and exogenously drawn from the distribution $G(w)$:

$$w_1, w_3 \quad \text{given} \tag{9}$$

$$w_2 \sim G(w) \tag{10}$$

The government wants to maximize its objective (1) subject to the government budget constraints (2)-(4), the laws of motion for employment (6)-(8) and the price schedule (5), taking wages (9)-(10) as given.

Investors are atomistic and risk-neutral, discounting future repayments in accordance with the risk-free interest rate r. The debt market is perfectly competitive such that lenders' maximization problem is

$$\max -q_1 B_2 + \mathbb{E}\left(\frac{d_2}{1+r}\right) B_2 \tag{11}$$

reflecting that they lend $q_1 B_2$ in the first period, but the amount B_2 is only redeemed with repayment probability $\mathbb{E}(d_2)$ and in the following period, which is discounted by $1/(1+r)$.

Timing. Given the initial state $s_0 = (w_1, w_3, N_1)$, the decisions of the agents are made according to the following order of moves (see Figure 1). In period $t = 1$, the government acts first by choosing taxes τ_1 and borrowing B_2. Then investors set debt prices q_1 to break even in expectation. Afterwards, next period's employment state N_2 is determined. At the beginning of period $t = 2$ wages w_2 materialize. The government defaults or repays d_2 and collects taxes τ_2 before next period's employment state N_3 is pinned down. Lastly, in period $t = 3$, the government decides on its tax policy τ_3.

The government is not able to commit to any future policies in an earlier period. Otherwise the government could directly commit to debt repayment ($d_2 = 1$) or it could promise to set high taxes in the second period, implying that defaulting would violate the budget constraint and thus indirectly commit not to default. This timing is in line with the standard Eaton and Gersovitz (1981) and Arellano (2008) models because the government chooses its new level of indebtedness before investors move in

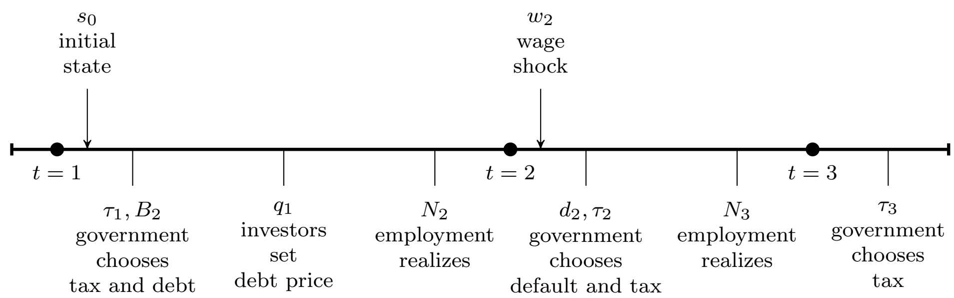

Figure 1: Timing of the model: The government moves before the investors in period $t = 1$, retaining its first-mover advantage.

the period in which the debt is priced. Multiplicity because of rollover risk or due to a price-taking behavior of the government is therefore not present in this model.

Recursive formulation. Since the government cannot commit to future policies the social welfare problem can be expressed recursively. Substituting out taxes using the budget constraint (2), borrowing costs according to the debt price schedule (5) and the employment law of motion (6), the expected value of the government in the first period is given by:

$$V^1(s_0) = \max_{B_2} \quad N_1 u\left(w_1 - \frac{(1-N_1)T - Q(B_2)B_2}{N_1}\right) + (1-N_1)u(T) + \beta \mathbb{E} V^2(s_0, w_2, H_2(N_1, Q(B_2), B_2), B_2) \tag{12}$$

More borrowing that leads to a higher capital inflow $Q(B_2)B_2$ means higher consumption in period $t = 1$ and higher employment levels in period $t = 2$,[4] but comes at the cost of a higher debt burden, entering the continuation value $V^2(\cdot)$ through the state B_2.

The value functions for repayment $V^{rep}(\cdot)$ and default $V^{def}(\cdot)$, among which the government chooses the higher to obtain the second period's value $V^2(\cdot)$, follow from including the government budget constraint (3) and the law of motion of employment

[4] This requires that $H_2(N_1, q_1, B_2)$ depends positively on the capital inflow $Q(B_2)B_2$, see the discussion below.

(6):

$$V^2(s_0,w_2,N_2,B_2) = \max\left\{V^{def}(s_0,w_2,N_2), V^{rep}(s_0,w_2,N_2,B_2)\right\} \tag{13}$$

$$V^{def}(s_0,w_2,N_2) = N_2 u\left(w_2 - \frac{(1-N_2)T}{N_2}\right) + (1-N_2)u(T) + \beta V^3(s_0, H_3(N_2,0)) \tag{14}$$

$$V^{rep}(s_0,w_2,N_2,B_2) = N_2 u\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right) + (1-N_2)u(T) + \beta V^3(s_0, H_3(N_2,1)) \tag{15}$$

The trade-off between default and repayment is a choice between higher consumption in period $t = 2$ and higher employment levels in period $t = 3$. If the former outweighs the latter, the government defaults ($d_2 = 0$) and accepts higher future unemployment through the law of motion of employment $H_3(N_2, 0)$. Otherwise the sovereign honors its debt obligations, which lowers consumption through higher taxes but ensures higher employment levels in the last period via $H_3(N_2, 1)$.

Lastly, the value function in the last period is given by:

$$V^3(s_0,N_3) = N_3 u\left(w_3 - \frac{(1-N_3)T}{N_3}\right) + (1-N_3)u(T) \tag{16}$$

The government is forced to set taxes in line with the budget constraint (4) but does not solve a maximization problem.

Definition 1 *Given an initial employment level N_1 and wages w_1 and w_3 summarized in s_0, a Markov-perfect equilibrium is a collection of*

- *government policies $\tau_1(s_0)$, $B_2(s_0)$, $\tau_2(s_0,w_2,N_2,B_2)$, $\delta(s_0,w_2,N_2,B_2)$, $\tau_3(s_0,N_3)$,*
- *value functions at each stage $V^1(s_0)$, $V^2(s_0,w_2,N_2,B_2)$, $V^{def}(s_0,w_2,N_2)$, $V^{rep}(s_0,w_2,N_2,B_2)$, $V^3(s_0,N_3)$, and*
- *a price schedule $Q(B_2)$,*

such that

1. *policies and associated value functions are optimal, i.e. debt issuance maximizes social utility and default occurs if the value of default exceeds the value of repayment,*
2. *policies are feasible, i.e. the tax policies satisfy the government budget constraints (2)-(4),*
3. *given government's policies, the price schedule makes investors break even in expectation, and*
4. *expectations over the wage w_2 are formed in line with (10) and the probability distribution $G(w)$, and employment follows given functions (7)-(8).*

Equilibrium characterization. To understand the origin of multiplicity in this model, it is useful to establish certain characteristics of the equilibrium. Since all value functions naturally depend on the initial condition $s_0 = (w_1, w_3, N_1)$, it is dropped from the state space for notational convenience from now onwards.

Optimality for the government can be described by backward induction. In the last period, the government has no choices to make as the tax rate is pinned down by the budget constraint and the value $V^3(N_3)$ only depends on employment N_3.

In the penultimate period, the government understands that employment N_3 evolves according to the function $H_3(N_2, d_2)$ and so that default affects the value $V^3(N_3)$. The default decision can be expressed as:

$$d_2 = \begin{cases} 0 & \text{if } V^{def}(w_2, N_2) > V^{rep}(w_2, N_2, B_2) \\ 1 & \text{else} \end{cases} \tag{17}$$

Alternatively, the condition for default $d_2 = 0$ is given by:

$$\begin{aligned} &N_2 u\left(w_2 - \frac{(1-N_2)T}{N_2}\right) + \beta V^3(H_3(N_2, 0)) > \\ &N_2 u\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right) + \beta V^3(H_3(N_2, 1)) \end{aligned} \tag{18}$$

The government's trade-off is concerned with the fact that defaulting lowers taxes and increases consumption for the employed workers in the second period, but imposes the cost of having higher unemployment and therefore more workers with a low benefit consumption level in the last period.

Condition (18) also illustrates that the default policy in equilibrium is a function of the state, $\delta(w_2, N_2, B_2)$. In particular, there is a default threshold in the wage w_2 such that if repayment is optimal at a certain wage, then repayment is also the dominant strategy if wages are higher. The reason is that although both values $V^{def}(w_2, N_2)$ and $V^{rep}(w_2, N_2, B_2)$ are increasing in the wage w_2, the value of repayment grows at a faster rate:

$$\frac{\partial V^{def}(w_2, N_2)}{\partial w_2} = N_2 u'\left(w_2 - \frac{(1-N_2)T}{N_2}\right) < \\ \frac{\partial V^{rep}(w_2, N_2, B_2)}{\partial w_2} = N_2 u'\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right) \tag{19}$$

The value of repayment increases by more at the margin because the consumption level is smaller and there is constant relative risk aversion. This means that if a wage $\hat{w}$ equalizes the default and repayment values in (18), then for any higher wage $w_2 > \hat{w}$ the value of repayment rises above the value of default and repayment is optimal, keeping indebtedness and unemployment constant. This implies that there is a unique threshold $\hat{w}$ for each pair (B_2, N_2), which allows to write the threshold $\hat{w}(N_2, B_2)$ as an implicit function of employment and debt, not a correspondence, indicating indifference between default and repayment:

$$N_2 u\left(\hat{w}(N_2, B_2) - \frac{(1-N_2)T}{N_2}\right) + \beta V^3(H_3(N_2, 0)) = \\ N_2 u\left(\hat{w}(N_2, B_2) - \frac{(1-N_2)T + B_2}{N_2}\right) + \beta V^3(H_3(N_2, 1)) \tag{20}$$

Similarly, if default is optimal at a given debt level, it will continue to dominate repayment for even higher liabilities. By increasing levels of indebtedness B_2, the value

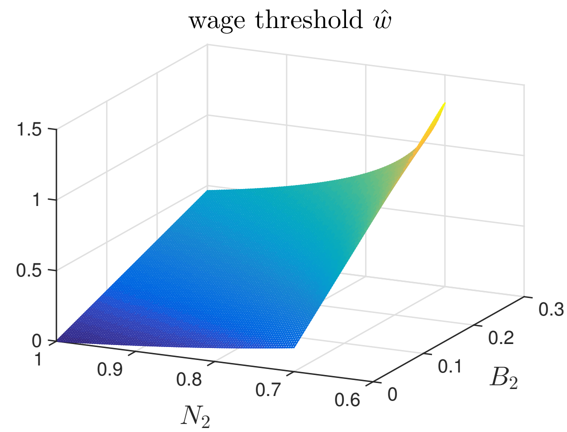

Figure 2: Wage threshold $\hat{w}(N_2, B_2)$: The threshold is a function of employment and debt, above which repayment is optimal. With monotonicity in debt and employment, it is decreasing in employment and increasing in debt.

of repayment falls while the value of default remains unchanged:

$$\frac{\partial V^{def}(w_2, N_2)}{\partial B_2} = 0 > \tag{21}$$

$$\frac{\partial V^{rep}(w_2, N_2, B_2)}{\partial B_2} = -u'\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right) \tag{22}$$

It exists therefore a debt threshold at each given wage and employment level above which default dominates repayment.

Lastly, I assume that the law of motion of employment $H_3(\cdot)$ is such that repayment is also more likely with higher employment. This monotonicity is a crucial property for the employment cost of default as described in Balke (2017). A sufficient condition for this to hold is that the value of default is non-increasing in employment N_2, while the value of repayment rises with employment:

$$\frac{\partial V^{def}(w_2, N_2)}{\partial N_2} \leq 0 \tag{23}$$

$$\frac{\partial V^{rep}(w_2, N_2, B_2)}{\partial N_2} > 0 \tag{24}$$

Equation (23) holds whenever the negative effect of default on the employment state $\frac{\partial H_3(N_2,0)}{\partial d_2} < 0$ is strong enough. Inequality (24) is satisfied if repayment does not destroy jobs, $\frac{\partial H_3(N_2,1)}{\partial d_2} \geq 0$. Appendix **??** discusses these conditions.

Figure 2 depicts a threshold function of the described form. For each pair of B_2 and N_2 there is a unique cut-off value of the wage w_2, above which debt is repaid. Given a share of employed workers N_2, the threshold is monotonically increasing in the debt level. The monotonicity of the default decision in N_2 ensures that $\hat{w}(N_2, B_2)$ is also monotonic in employment. Figure 2 illustrates that $\hat{w}(N_2, B_2)$ decreases in employment because default becomes more costly and thus the indifference wage level shrinks ceteris paribus.

In period 1, the optimal debt issuance solves:

$$\begin{aligned}
\max_{B_2} N_1 u\left(w_1 - \frac{(1-N_1)T + Q(B_2)B_2}{N_1}\right) + (1-N_1)u(T)+ \\
\beta \mathbb{E} \max\left\{V^{def}(w_2, H_2(N_1, Q(B_2), B_2)), V^{rep}(w_2, H_2(N_1, Q(B_2), B_2), B_2)\right\} \qquad (25)
\end{aligned}$$

The government forms expectations over w_2, takes into account the law of motion for employment $H_2(\cdot)$ from (6) and chooses borrowing B_2. Using the threshold property of default at $\hat{w}(N_2, B_2)$, the problem can be rewritten as

$$\begin{aligned}
&\max_{B_2} N_1 u\left(w_1 - \frac{(1-N_1)T + Q(B_2)B_2}{N_1}\right) + (1-N_1)u(T)+ \\
&\beta \int_{\underline{w}}^{\hat{w}(H_2(N_1,Q(B_2),B_2),B_2)} V^{def}(w_2, H_2(N_1, Q(B_2), B_2))dG(w_2)+ \qquad (26) \\
&\beta \int_{\hat{w}(H_2(N_1,Q(B_2),B_2),B_2)}^{\overline{w}} V^{rep}(w_2, H_2(N_1, Q(B_2), B_2), B_2)dG(w_2) \qquad (27)
\end{aligned}$$

and results in the first order condition for debt issuance[5]

$$\begin{aligned}
&u'(\cdot)\left[Q'(B_2)B_2 + Q(B_2)\right] = \\
&\beta \int_{\underline{w}}^{\hat{w}(\cdot)} \frac{\partial V^{def}(\cdot)}{\partial N_2}\left(\frac{\partial H_2(\cdot)}{\partial Q(B_2)}\frac{\partial Q(B_2)}{\partial B_2} + \frac{\partial H_2(\cdot)}{\partial B_2}\right) dG(w_2)+ \qquad (28) \\
&\beta \int_{\hat{w}(\cdot)}^{\overline{w}} \frac{\partial V^{rep}(\cdot)}{\partial B_2} + \frac{\partial V^{rep}(\cdot)}{\partial N_2}\left(\frac{\partial H_2(\cdot)}{\partial Q(B_2)}\frac{\partial Q(B_2)}{\partial B_2} + \frac{\partial H_2(\cdot)}{\partial B_2}\right) dG(w_2) \qquad (29)
\end{aligned}$$

[5]Note that the first order condition is obtained using the Leibniz rule, however, due to the fact that the default threshold also changes with B_2, terms cancel out.

which shows that the optimal borrowing is a function of the debt price schedule $Q(\cdot)$.

In equilibrium, the debt price schedule $Q(\cdot)$ has to be consistent with optimality on the part of competitive and risk-neutral lenders. Lenders' maximization problem requires that the optimal debt price q_1 in (11) satisfies

$$q_1 = \mathbb{E}\left(\frac{d_2}{1+r}\right) \tag{30}$$

i.e. that it equals the default-risk adjusted inverse of the gross interest rate. Equation (30) is a zero-profit condition such that competitive investors break even in expectation. From (5), the pricing $Q(B_2)$ can only be an equilibrium price schedule if:

$$Q(B_2) = \mathbb{E}\left\{\frac{d_2}{1+r}\right\} \tag{31}$$

Investors anticipate the government's default decision and price the debt accordingly.

3.2 Multiplicity with dynamic private sector behavior

The model is able to produce multiple equilibria. Two features of the newly introduced employment state are important for the emergence of this three-state multiplicity.

First, default is more likely when unemployment is high and so the default decision is a function of the employment state N_2:

$$d_2 = \delta(w_2, N_2, B_2) \tag{32}$$

This is the standard Markov structure of the government policy. For the default decision to meaningfully react to changes in employment it is sufficient that the partial derivatives of the values of default and repayment satisfy (23)-(24). As shown above, the wage threshold $\hat{w}(N_2, B_2)$ is then monotonically decreasing in the unemployment rate. In particular, as the cost of default materialize in terms of employment drops and the model features asymmetries, the default policy reacts elastically to changes in employment $\partial\delta/\partial N_2 \geq 0$. Incorporating the default function (32) in the zero-profit equation (31), it prescribes that in equilibrium:

$$Q(B_2) = \mathbb{E}\left\{\frac{\delta(w_2, N_2, B_2)}{1+r}\right\} \tag{33}$$

Investors form rational expectations over w_2 and observe B_2. In addition, the risk-neutral pricing of bonds is also a function of tomorrow's employment level, so investors must form expectations over N_2.

Second, employment is a function of the debt price and privately determined through the law of motion (7), which can be rewritten as:

$$N_2 = H_2(N_1, Q(B_2), B_2) \tag{34}$$

This reduced-form evolution equation reflects that debt prices affect the pre-financing of vacancies and wages. After replacing employment with the law of motion in the bond pricing function (33), the zero-profit condition for the investors reads:

$$Q(B_2) = \mathbb{E}\left\{\frac{\delta(w_2, H_2(N_1, Q(B_2), B_2), B_2)}{1+r}\right\} \tag{35}$$

The price schedule $Q(B_2)$ appears on both sides of equation (35), which offers the possibility for more than one solution to it. In other words, there may be multiple price schedules $Q(B_2)$ that are consistent with an equilibrium, i.e. multiple equilibria. Notably, since this is the case for any given B_2, there may be multiple equilibria despite the first-mover advantage of the government.

The intuition for the origin of multiplicity is the following: If investors expect high unemployment tomorrow and thus default, this can become self-fulfilling because they set low debt prices. By the employment evolution equation low debt prices lead to high unemployment, validating the pessimistic expectations of the investors. But since default is more likely with high unemployment, the low debt prices were also justified. In contrast, high expected employment tomorrow implying low default probabilities are self-fulfilling, too, because good prices imply high employment in the following period and so a low propensity to default.

Examples. Multiple equilibria are present in this model if for a state s_0 and after the government has moved first setting B_2, there are at least two distinct prices q_1 that are consistent with equilibrium. Simple functions for the laws of motion of employment, $H_2(\cdot)$ and $H_3(\cdot)$, can serve as examples for the model mechanism that allows for multiplicity of equilibrium.

As a first example consider the case with only two employment levels in the second period, high N_2^h and low N_2^l, and with a cut-off value for the debt price q_1, above which the high employment level is reached and the low level otherwise:

$$H_2(N_1, q_1, B_2) = \begin{cases} N_2^h & \text{if } q_1 \geq \hat{q} \\ N_2^l & \text{else} \end{cases} \tag{36}$$

Further assume that employment flows in (8) adopt the functional form

$$H_3(N_2, d_2) = \begin{cases} N_3^h & \text{if } d_2 = 1 \\ N_3^m & \text{if } d_2 = 0 \quad \wedge \quad N_2 = N_2^l \\ N_3^l & \text{else} \end{cases} \tag{37}$$

This law of motion ensures that (23)-(24) hold such that the government has a higher incentive to default in times of high unemployment in reference to the monotonicity of the repayment decision in employment $\delta(w_2, N_2, B_2)$.

This example serves the purpose of showing that investors' sentiments play a significant role and that expectations about default probabilities are linked to expectations about the third state variable. Investors either expect employment in the next period to be high, which by (36) and (33) imply a high debt price q_1, or they anticipate a high unemployment rate instead, in accordance with a low price. If the threshold $\hat{q}$ in (36) lies between these two implied price levels, expectations about employment and default are self-fulfilling and there are two distinct solutions to the investors' problem. Therefore, two distinct price schedules $Q(B_2)$ can be found that satisfy equilibrium conditions corresponding to two equilibria.

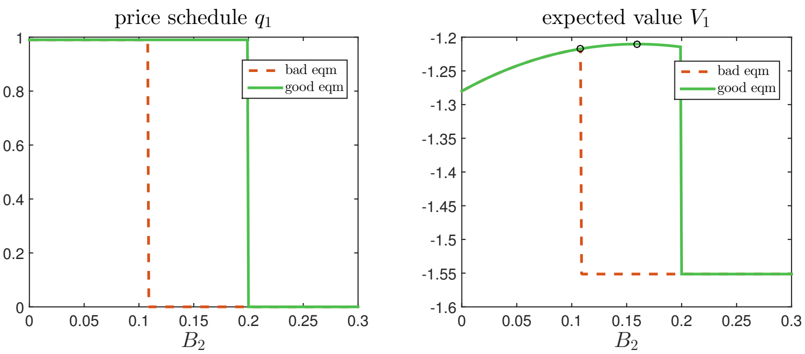

Figure 3: Multiple equilibria: Equilibrium price schedules (left) and expected first-period values (right) as a function of debt issuance. Circles indicate that the good equilibrium (solid green line) attains a higher value than the bad equilibrium (dashed red line). Values: $\beta = 0.95, r = 0.01, \sigma = 1, T = 0.4, w_1 = 0.5, w_2 = 0.8, w_3 = 0.8, N_1 = 0.92, N_2 \in \{0.85, 0.95\}, N_3 \in \{0.72, 0.83, 0.97\}, \hat{q} = 0.5$.

Figure 3 (left) plots debt issuance B_2 together with equilibrium price schedules $Q(B_2)$. On the one hand, for very low debt levels B_2 default is too costly and only good debt prices are consistent with equilibrium. On the other hand, for large amounts of debt it is always optimal for the government to default, so debt prices fall to zero in equilibrium to satisfy free entry of investors. However, in a medium borrowing range there exist one good and one bad price schedule, the former implying higher debt prices and lower borrowing costs for the government (solid green line) while the other features worse prices (dashed red line). This is the range of debt levels, for which multiplicity is possible.

Of course, this would not be an issue if the borrower did not want to sell debt in the medium range, e.g. if the government could attain maximum welfare by borrowing small amounts to the left of the multiplicity region. However, Figure 3 (right) plots the corresponding values for social welfare in the first period $V_1(B_2)$. Note that there are two distinct value functions corresponding to the two price schedules. The circles indicate social optima for the economy when facing either pessimistic or optimistic investors, respectively, and the implied levels of debt on the x-axis. Since the overall maximum is attained in the multiplicity region for debt conditional on low borrowing costs, the possibility of a debt crisis due to self-fulfilling default fears is a concern.

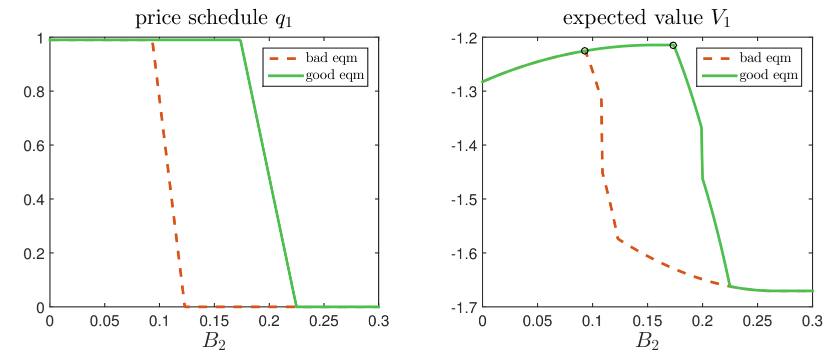

Figure 4: Multiple equilibria with uncertainty: Equilibrium price schedules (left) and expected first-period values (right) as a function of debt issuance. Debt price schedules fall gradually with uncertainty. Values: $\beta = 0.95, r = 0.01, \sigma = 1, T = 0.4, w_1 = 0.5, w_2 \sim [0.7, 0.9], w_3 = 0.8, N_1 = 0.92, N_2 \in \{0.85, 0.95\}, N_3 \in \{0.72, 0.83, 0.97\}, \hat{q} = 0.5$.

In a situation where a government issues the optimal amount of bonds assuming a coordination on good debt prices but ends up facing much higher borrowing costs and unemployment rates, the implications of expectation-driven crises appear substantial.

Multiplicity is not present on account of a particular distribution of the exogenous state variable. In fact, the distribution of wage w_2 does not play a role in this example because it is fixed at a certain level. Without uncertainty the government in the first period knows exactly when default is optimal and strategically chooses to borrow only up to the point where it still receives positive debt prices. However, this means that with the adverse debt schedule, the government chooses less debt and the value is smaller (left circle) – compared to the good debt schedule (right circle) – and issuing more debt would not be optimal as above this level borrowing costs would be infinite.

In the second example, wage w_2 is stochastic and follows a uniform distribution, $w_2 \sim [\underline{w}, \overline{w}]$. Figure 4 plots the implied price correspondence (left) and first-period values (right) against the number of bonds issued. The laws of motion of employment are pinned down by the same functions as in the previous example. The main difference is that due to the stochastic nature of the wage, the price schedules do not include any vertical parts, reflecting the fact that the default decision also depends on the realization of wages. Prices $Q(B_2)$ still (weakly) decrease in debt issuance B_2, but for

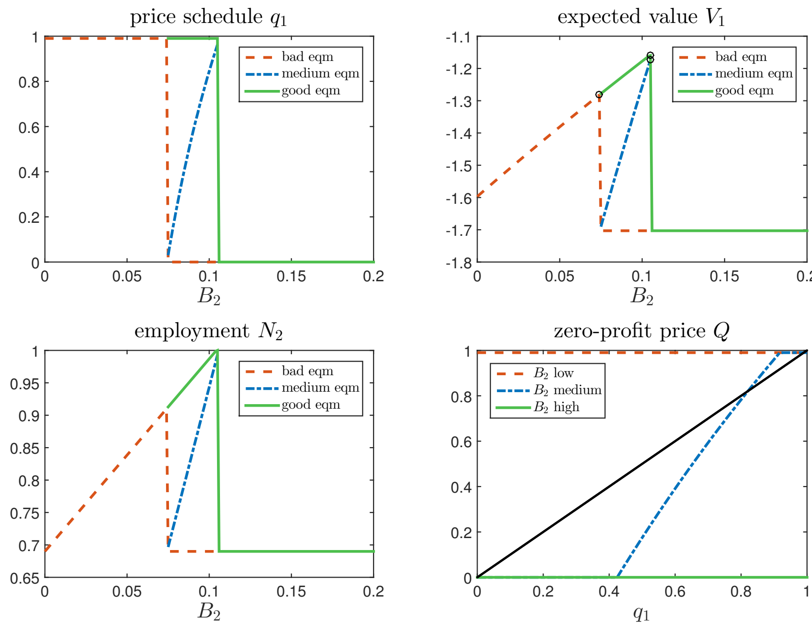

Figure 5: Multiple equilibria with three price schedules: Equilibrium price schedules (top left), expected first-period values (top right) and employment (bottom left) as a function of debt issuance. Initial price and implied zero-profit price for several debt levels (bottom right). Values: $\beta = 0.95, r = 0.01, \sigma = 1, T = 0.4, w_1 = 0.5, w_2 \sim [0.7, 0.9], w_3 = 0.8, N_1 = 0.92, a_1 = 0.75, a_2 = 3, N_3^l = 0.72, N_3^m = 0.83, N_3^h = 0.97$.

medium ranges of debt the possibility of receiving a high wage draw is responsible for the gradual fall of the price schedules. As before, the government chooses the debt level that achieves the maximum expected value in the first period, but achieves a lower value if it is confronted with a bad price schedule (Figure 4, right).

In a third example, the law of motion for employment $H_2(\cdot)$ is given by:

$$H_2(N_1, q_1, B_2) = a_1 N_1 + a_2 q_1 B_2 \tag{38}$$

In contrast to the previous law of motion, second-period employment is now a function of not only q_1 but also N_1 and B_2.[6] Employment in the third period evolves according

[6]The inclusion of the product of the price and debt level into the functional form is chosen in accordance with the detailed model in Balke (2017).

to

$$H_3(N_2, d_2) = \begin{cases} N_3^h & \text{if } d_2 = 1 \\ N_3^m - (N_3^m - N_3^l)N_2 & \text{if } d_2 = 0 \end{cases} \tag{39}$$

where unemployment is low if the government repays but otherwise increases in the previous period's employment level to ensure monotonicity in N_2.

While the other examples have already shown that a dependence of the employment state on the debt price q_1 can suffice for multiplicity, this example is designed to illustrate that there may be more than two price schedules. Figure 5 plots prices as a function of debt issuance B_2 (top left). In addition to a good and bad schedule, a third schedule emerges in between the other two (blue dashed-dotted line). For this medium price schedule, the debt price is *increasing* in the amount of debt issued and also the corresponding value (top right) is *increasing* over this debt range. The government thus chooses more rather than less debt when confronted with it. The reason is that with a higher price q_1 also employment N_2 increases as shown in the bottom left panel. A high debt price and debt issuance increases employment, making debt more sustainable in the next period and thus justifying the high debt price in the first place.

The bottom right panel of Figure 5 plots an arbitrary initial debt price q_1 against the debt price that is implied by the optimal default decision Q_1, for three different levels of sovereign borrowing. The 45 degree line indicates debt prices that are consistent with equilibrium behavior. At very low debt levels, the implied debt price only crosses once at a high debt price, which is the unique equilibrium price (red dashed line). In contrast, for a high debt exposure, prices fall to zero with a unique crossing of the 45 degree line at this point (green solid line). However, in a middle range the implied price schedule is upward sloping, giving rise to three crossings and equilibrium prices (blue dashed-dotted line). This shows that at medium debt levels, multiple equilibria can arise. Since the pricing function crosses from below the medium debt price equilibrium is unstable in the sense that if in an auction a slightly higher price was initially announced the (too little) default behavior should push the price up to a high price, and in reverse for a smaller initial candidate price the ultimate debt price would go down to zero.

What these examples show is that multiplicity originates from the presence of another state variable that is determined by the debt price and influences default behavior. The stochastic process influences the shape of the price schedules but is not the source of multiplicity. Furthermore, the exact functional form of the evolution of the new state variable has a crucial impact on the shape and number of price schedules.

Lastly, the attained social values by a government when facing worse debt schedules also decline. Policies that aim at shifting the economy into a better equilibrium with lower borrowing costs may therefore be welfare-improving. It is however an open question how the government can do this and if announcements to guarantee certain debt repayments requires supranational institutions with more commitment power (see Section 5).

3.3 Relation to other sources of multiplicity

This section gives an overview of other ways through which multiple equilibria can exist in sovereign default models in order to discuss how those sources of multiplicity differ from the one described in this paper. Since most default models only have two state variables – one exogenous state of endowment and one endogenous state of debt – I begin revisiting sources of multiplicity in two-state models. Then I show why including employment as a third state variable gives opportunity for multiple equilibria to emerge.

Multiplicity in two-state default models. The model in Arellano (2008) has a unique equilibrium. The government announces default or repayment on currently outstanding debt and issues new debt at maturity before creditors price it according to their zero-profit condition. The government has a first-mover advantage: After the exogenous endowment state w realizes at the beginning of the period, the government issues bonds B' facing a pricing schedule $Q(B', w)$, which only depends on the already observed current state and its own debt choice. When the government makes its borrowing decision there is therefore no uncertainty about the subsequent pricing of investors because the timing allows the government to choose a specific point on this debt price schedule. It satisfies that investors break even in equilibrium

$$Q(B', w) = \mathbb{E}\left\{\frac{\delta(w', B')}{1+r}\right\} \tag{40}$$

but no other actions are taken until the next period when the government again chooses to default or repay $\delta(w', B')$, which is a function of next period's endowment w' and the chosen debt level B'. The rational sovereign understands that the debt price schedule in (40) is directly and uniquely determined by its issuance of new debt B'. The proof of uniqueness in Auclert and Rognlie (2014) shows that since the government can set the debt level first and mimic any other government with potentially higher debt, it can always achieve the maximum by choosing the optimal debt level.[7] They also rule out multiplicity of the form where bond prices in the good equilibrium dominate those in a self-fulfilling adverse one with stochastic reentry, a common assumption in the default literature.

Other two-state default models broke the uniqueness result by departing from the standard timing assumption. This disables the sovereign from choosing the most favorable point on the debt schedule and can give rise to more than one price schedule consistent with equilibrium behavior.

First, models with rollover risk as in Cole and Kehoe (2000) feature multiple equilibria that arise because new bond issuance takes place before the default decision. In the good equlibrium, the government can issue new debt at good prices which is used to repay old debt justifying the high debt prices since no default occurs. In the bad equilibrium, the government cannot rollover debt due to low debt prices such that it defaults, making the low debt prices consistent with equilibrium default. The default decision is a function of the revenue from debt issuance and thus today's debt price is

[7] Even if there were worse combinations of debt issuance and prices that satisfied investors' optimality, the government can avoid such a worse outcome. In fact, since in equilibrium the government must also behave optimally, there is only one equilibrium.

a function of tomorrow's debt price:

$$Q(B', w) = \mathbb{E}\left\{\frac{\delta(w', B'', Q(B'', w'))}{1+r}\right\} \tag{41}$$

In a recursive equilibrium, the price schedule has to be constant over time and thus operates on both sides of the pricing equation with the consequence that there may be multiple solutions. This rollover multiplicity is ruled out in this paper because the default decision takes place at the beginning of the period.

Second, multiple equilibria in the tradition of Calvo (1988) arise when the government faces a Laffer curve in its ability to raise revenue. When the government chooses current debt instead of debt at maturity, the amount of borrowing required to raise a given revenue depends on the endogenous interest rate (Lorenzoni and Werning (2014)). If endogenous interest rates are low, the government needs to issue more debt to collect the same amount of revenue, which increases next period's default probability and justifies the low interest rates. The interest rate schedule becomes a function of current debt B, rather than debt at maturity, while the default decision depends on outstanding debt including interest:

$$Q(B, w) = \mathbb{E}\left\{\frac{\delta(w', B/Q(B, w))}{1+r}\right\} \tag{42}$$

Ayres, Navarro, Nicolini, and Teles (2015) show that with a bimodal shock distribution multiple equilibria are still possible even with debt at maturity as long as the bond price is set first by creditors. One can easily replace current debt B by debt at maturity B' and there is still more than one solution to equation (42). The important assumption for the Laffer curve multiplicity is thus the timing, according to which creditors set a price before bonds are sold by the sovereign. In this paper, the government chooses bonds first (and issues debt at maturity) ensuring that it always picks the good side of the Laffer curve which rules out Calvo type multiplicity.

The common departure from the standard setup in Cole and Kehoe (2000) and Lorenzoni and Werning (2014) is the change in the order of moves such that the government has the possibility to react to failed auctions: either by defaulting if the revenue is insufficient to rollover old debt or by selling more bonds which dilutes debt. This paper takes a different route by keeping the same timing assumption as in the benchmark, which widely rules out multiple debt price schedules, but still proves multiplicity by introducing employment as a third state variable.

Three-state multiplicity. Multiplicity due to a third state variable can emerge despite the standard Arellano (2008) timing if – as in the model outlined above – the model features an equilibrium condition of the form

$$Q(B', w) = \mathbb{E}\left\{\frac{\delta(w', H(Q(B', w), B'), B')}{1+r}\right\} \tag{43}$$

Importantly, the debt price schedule $Q(\cdot)$ is present on both sides of the equation with the consequence that there may be more than one possible solutions for any borrowing B'. This is true even if debt is issued before the price is set, i.e. if the timing assumption is standard.

However, there are two possible ways of how the model can be changed such that multiplicity is ruled out, namely if the third state variable is either independent of the debt price or contractible. First, if employment is not influenced by the debt price, equation (43) would instead read

$$Q(B', w) = \mathbb{E}\left\{\frac{\delta(w', H(B'), B')}{1+r}\right\} \tag{44}$$

because the debt price does not enter the law of motion for employment any longer. Now, the price is unambiguously determined by the debt issuance B' as everything else in known or follows an exogenous process. Second, it would also matter if the government could promise a certain employment level at the same time as making its debt choice, i.e. if the employment state became contractible. In this case, the government sets a particular pair of policies for future debt and employment (B', N'), both of which are known at the stage of debt pricing:

$$Q(B', N', w) = \mathbb{E}\left\{\frac{\delta(w', N', B')}{1+r}\right\} \tag{45}$$

Now the price schedule $Q(\cdot)$ is directly pinned down in this case as well, but it becomes a function of the future employment level. It is questionable however, how the government can deliver a specific unemployment rate in the future and influence a state that is widely determined in the private sector. In addition, even if it could, the government would be assigned partial commitment power by the implicit assumption that it would not only promise a specific state but also actually implement these promised employment states. A contractible additional state changes the nature of the commitment problem and is of no use to explain how expectations can affect the debt pricing because if the government can single-handedly set future employment levels, expectations are ruled out to have an effect from the start.

A particular structure of the zero-profit condition must therefore be kept in order to ensure the possibility of multiple equilibria of this type. Similarly to the previous equilibrium conditions in two-state models with more than one possible solutions, here the zero-profit condition has to hold and pins down possibly more than one debt price. However, the difference is that the government chooses debt at maturity and more importantly, that here the debt price is set after the government moved.

4 Dynamic labor market

The previous discussion has identified the crucial features of employment that are responsible for the occurrence of multiple equilibria. This section provides a microfoundation that can rationalize these features, especially why the evolution of employment depends on the debt price. The mechanism relies on vacancy-posting firms that borrow to pre-finance vacancies and pay interest rates, which co-move with sovereign default spreads. This is in line with empirical research finding a transmission of sovereign risk to the private lending conditions (Acharya, Eisert, Eufinger, and Hirsch (2015), Gilchrist and Mojon (2014) and Adelino and Ferreira (2016)). The idea is that if sovereign bond prices fall, this increases private borrowing rates and thus depresses vacancy postings.[8]

The reduced-form laws of motion of employment used above can be microfounded as in the standard frictional labor market models in the tradition of Diamond, Mortensen and Pissarides.[9] Then next period's employment N' is pinned down by the previous firm-worker matches N that are not destroyed with rate ξ and new matches $M(1-N,v)$ as a function of current unemployment and vacancies:

$$N' = (1-\xi)N + M(1-N,v) \tag{46}$$

Let $M(\cdot)$ be a standard Cobb-Douglas matching function

$$M(1-N,v) = \mu(1-N)^{\psi}(v)^{1-\psi} \tag{47}$$

where μ governs match efficiency and ψ the elasticity of matches to unemployment. Furthermore, let $\mathcal{J}(\cdot)$ be the value of a firm with a filled job, a be the cost of a vacancy and $1/(1+r)$ be the discount factor, then free entry imposes the following equilibrium condition:

$$(1+R)a = \lambda_f(1-N,v)\frac{1}{1+r}\mathbb{E}\{\mathcal{J}(N')\} \tag{48}$$

It prescribes that the cost of a vacancy has to be equal to the expected probability of being filled multiplied by the discounted future value of a job. The vacancy filling rate follows as the ratio of matches to total vacancies, $\lambda_f(\cdot) = M(\cdot)/v$. The pre-financing condition of vacancies shows up in the interest rate R. A firm's expected future value depends on the aggregate state, including employment and possibly other states (here suppressed for notational convenience), $\mathcal{J}(N')$.

On the basis of these equations one can show how multiple equilibria can arise. Since default is more likely with high unemployment, investors who expect high un-

[8]Balke (2017) provides a detailed model and also considers the possibility of firing due to excessive interest rate increases but the channel is not dependent on this assumption.

[9]See their seminal papers Diamond (1982b), Diamond (1982a), Mortensen (1982a), Mortensen (1982b), Pissarides (1979), Pissarides (1985), Mortensen and Pissarides (1994) and a long list of work that followed.

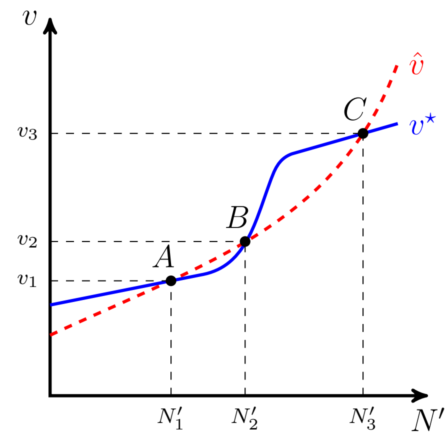

Figure 6: Multiplicity of equilibria: Vacancies consistent with matching (red dashed line) and free entry (blue solid line) given next period's employment level.

employment set low debt prices and vice versa, captured by $q(N')$. The co-movement of private and sovereign spreads imply the relationship $R(q)$.[10] Then let $v^\star$ be the vacancy level that satisfies free entry

$$(1 + R(q(N')))a = \lambda_f(1 - N, v^\star)\frac{1}{1 + r}\mathbb{E}\{\mathcal{J}(N')\} \tag{49}$$

such that a lower N' depresses vacancies $v^\star$. At the same time, the law of motion needs to be satisfied, so for a given N' let $\hat{v}$ be the number of vacancies that is consistent with equilibrium dynamics

$$N' = (1 - \xi)N + \mu(1 - N)^\psi(\hat{v})^{1-\psi} \tag{50}$$

where $\hat{v}$ is also increasing in N'.

There may be more than one level of vacancies that satisfies both the aggregate law of motion and the free entry condition $v = \hat{v} = v^\star$. Figure 6 plots the implied values for vacancies from (49) and (50) for each given level of N'. It shows that $\hat{v}$ is convex in N' which follows from the Cobb-Douglas matching function (red dashed line). In contrast, $v^\star$ can be S-shaped in N' (blue solid curve), which creates multiple

[10]Specifically, given that a change in tomorrow's employment N' affects the default probability it also alters the interest rate R through its effect on private lending and the expected value of a filled job.

intersections of $\hat{v}$ and $v^\star$.

It may be sufficient that the value of either the interest rate or the expected value of a job changes with N' to give $v^\star$ its S-shape, e.g. even for a fixed R uniqueness is not guaranteed. However, it can be shown that multiplicity is more likely when the value of a job differs substantially along with employment as the repayment status switches and when the interest rate reacts to the debt price q. In this case the number of vacancies $v^\star$ tilts a lot in the region of elastic default probabilities which is thus responsible for the convexity.

Intuitively, if investors expect a high propensity to default and set low debt prices the low debt price pushes private interest rates up and lead to low vacancies that satisfy free entry. This effect is amplified by a lower expected job value, which is attached to higher unemployment. These low vacancies can furthermore be consistent with the aggregate law of motion so that an equilibrium with low vacancies and high unemployment emerges. An elevated unemployment rate means that the possibility of generating revenue is lower yielding a higher default probability and justifying the initial low debt price. In contrast, high expected employment tomorrow implying low default probabilities are self-fulfilling, too, because high prices lead to low interest rates in the private sector, more vacancies and high employment.

5 Policy options

Since expectation-driven crises are associated with lower social welfare, it is important to understand what policy options are available to the sovereign in order to avoid being trapped in a bad equilibrium. I consider supranational lending and guarantees, alternative commitment structures and price floors.

First, in contrast to other types of multiplicity a lender of last resort is unlikely to be useful in preventing a bad coordination outcome. The reason is that the bad equilibrium is not a matter of an inability to sell debt as for example with rollover multiplicity, but rather that the low debt price resulting in the sovereign debt auction affects employment. What is needed is therefore a guarantee that debt is repaid, not that debt is issued. A supranational institution that declares such a guarantee – if large enough to rule out the bad equilibrium – would in fact never actually have to step in as the promise alone is sufficient for a better creditor coordination. This provides a rational for why guarantees by an international agency to buy back debt can be successful even if they are never actually called in, but a lender of last resort that can resolve problems when confronted with a rollover crisis may be less welfare-enhancing.

Second, three-state multiplicity is ruled out by giving the government commitment power to the private sector outcome, i.e. by making employment contractible. However, such an assumption is not only strong but also rather unrealistic as firms and workers determine labor market outcomes and it is questionable how a government could have the power and commitment ability to implement a promised unemployment rate. Since standard models of fundamental default show significant improvements by enhancing the commitment ability of the sovereign by increasing the default cost, a more plausible alternative may be to consider higher employment cost of default. However, although this could move the multiplicity region outwards as higher costs make default only optimal at larger levels of indebtedness, the fact that multiple equilibria exist is unchanged.

Third, one can consider price floors that force the debt price to be above a certain level. In this case, imagine the government wants to buy debt that would otherwise have a zero price as it lies in the region where all schedules reflect a certain default. Then lenders would refuse to buy the bonds but there would be no harm done because without the price floor the government would also choose not to issue debt at zero prices. Similarly, if the equilibrium price is above the floor it would be unconstrained without the regulation having bite. However, in the middle region where distinct equilibrium price schedules exist, a lower bound on prices may help the investors to coordinate on the good price schedule.

6 Conclusion

The theoretical model in this paper illustrates a novel source of multiple equilibria in sovereign default models. It can help to understand how expectations, in addition to fundamentals, can make an economy vulnerable to debt crises and what type of policies can prevent pessimistic expectations in a market panic from becoming self-fulfilling.

The model departs from the standard framework, which features a unique equilibrium, by introducing employment as an additional and privately determined state. The mechanism behind the expectations-driven movements in debt prices is that if investor's expect default and set high risk premia, these drive up unemployment which in turn makes default more likely and validates the lenders' original fears. Two features of the employment state are important in order for self-fulfilling multiple equilibria to arise. First, the optimality of default depends on the unemployment rate. Second, debt prices impact on the evolution of employment. The paper shows that on these two conditions multiplicity emerges with neither changing the standard first-mover advantage of the government in the game with investors nor its limited commitment.

Belief-driven debt crises are not a mere theoretical curiosity but a heavily debated symptom of the recent market panics in Europe. The policy insights of this paper are that policy makers, who want to react to the possibility of being caught in a bad market equilibrium with low welfare, may be able to successfully break bad expectations with repayment guarantees from supranational agencies or fixed floors on debt prices.

Lastly, the paper paves the way for a joint approach to study fundamental and expectation-driven debt and default crises because the theory I provide preserves the standard timing assumption of fundamental default models while simultaneously outlining three-state multiplicity as a way of incorporating a role for expectations. I hope this work helps future research to build a unified quantitative framework to make well-informed policy recommendations.

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A Threshold property of default

This is to show that there exists a default punishment in form of employment losses $\frac{\partial H_3(N_2,0)}{\partial N_2} < 0$ such that the value of default is non-increasing in employment levels in the second period N_2, while the value of repayment is increasing in N_2. First, consider the marginal value of default for a change in current employment levels:

$$\begin{aligned}\frac{\partial V^{def}(w_2,N_2)}{\partial N_2} =& u\left(w_2-\frac{(1-N_2)T}{N_2}\right)+u'\left(w_2-\frac{(1-N_2)T}{N_2}\right)\frac{T}{N_2^2}-u(T)+\\ &\beta\Bigg\{\Bigg[u\left(w_3-\frac{(1-H_3(N_2,0))T}{H_3(N_2,0)}\right)+\\ &u'\left(w_3-\frac{(1-H_3(N_2,0))T}{H_3(N_2,0)}\right)\frac{T}{H_3(N_2,0)^2}-u(T)\Bigg]\frac{\partial H_3(N_2,0)}{\partial N_2}\Bigg\}\\ \leq& 0 \end{aligned} \tag{51}$$

Note that for reasonable parameter values, the consumption level of the employed always exceeds the benefit level, $u(w-\tau)-u(T)>0$. I only consider this case, otherwise all employed workers could quite their jobs and enjoy higher consumption during unemployment. Then, the term in square brackets is positive as the marginal value of utility is strictly positive, too. However, since this term is multiplied by $\frac{\partial H_3(N_2,0)}{\partial N_2}$, one can always find a strong enough punishment $\frac{\partial H_3(N_2,0)}{\partial N_2} < 0$ that renders the overall term negative.

Turning to the value of repayment, note that the derivative with respect to employment N_2 is given by:

$$\begin{aligned}\frac{\partial V^{rep}(w_2, N_2, B_2)}{\partial N_2} =& u\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right) \\ &+ u'\left(w_2 - \frac{(1-N_2)T + B_2}{N_2}\right)\frac{T + B_2}{N_2^2} - u(T) + \\ &\beta\left\{\left[u\left(w_3 - \frac{(1-H_3(N_2,1))T}{H_3(N_2,1)}\right) + \right.\right. \\ &\left.\left. u'\left(w_3 - \frac{(1-H_3(N_2,1))T}{H_3(N_2,1)}\right)\frac{T}{H_3(N_2,0)^2} - u(T)\right]\frac{\partial H_3(N_2,1)}{\partial N_2}\right\} \\ >& 0 \end{aligned} \tag{52}$$

The terms in the first two rows as well as the term in square brackets are positive for the same reason as above. Then the entire right side of the equation is also positive if $\frac{\partial H_3(N_2,1)}{\partial N_2} > 0$, i.e. if there is no punishment from repayment.